Letting \( A \) be the \( 3N_b \times 3N_b \) matrix of entries

\[
A_{\alpha u}^{\beta w}(q) = \sum_{n=1}^{N_c} d_{\alpha u}^{\beta w}(l_m - l_n) \exp[jq \cdot (l_n - l_m)],
\]

which is obviously Hermitean, yields

\[
\sum_{\beta w} A_{\alpha u}^{\beta w}(q) \zeta_{\beta w}(0) = \omega^2 \zeta_{\alpha u}(0).
\]

For a given \( q \), the above is an eigenvalue equation of order \( 3N_b \), whose \( 3N_b \) eigenvalues are found by solving the algebraic equation

\[
\det(A - \omega^2 I) = 0.
\]

As the entries of \( A \) depend on \( q \), the calculation of the \( 3N_b \) eigenvalues must be repeated for each distinct value of \( q \), namely, \( N_c \) times. The total number of eigenvalues thus found is \( 3N_b \times N_c = 3N \), as should be.

The above result shows that, while the translational invariance eliminates the dependence on \( l_m \), it introduces that on \( q \). As the number of different determinations of the two vectors is the same, namely, \( N_c \), the total number of eigenvalues is not affected.