AN INTRODUCTION TO
WAVELET
TRANSFORMS

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OUTLINE

▪ Time-Frequency Analysis
▪ Introduction on Wavelet Operators
▪ Examples of applications: Radar/Sonar
▪ Experimental results
▪ Conclusions
Fourier Analysis

\[ F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j \omega t} dt \]

\[ f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j \omega t} d\omega \]

- Fast Discrete Algorithm (FFT)
- FFT: a rotation in function space
- New basis functions sines and cosines
- Not localized in time
Signal Analysis

\[ f(t) = f_1(t) + f_2(t) + f_3(t) \]

\[ f_2(t) = \sin \left( 2\pi \frac{t - 100}{T_2} \right) e^{-\left( \frac{t-100}{1.8T_2} \right)^2} \quad T_2 = 14 \]

\[ f_1(t) = \sin \left( 2\pi \frac{t - 30}{T_1} \right) e^{-\left( \frac{t-30}{2T_1} \right)^2} \quad T_1 = 28 \]

\[ f_3(t) = \sin \left( 2\pi \frac{t - 155}{T_3} \right) e^{-\left( \frac{t-155}{3.2T_3} \right)^2} \quad T_3 = 7 \]
Fast Fourier Transform

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Amplitude vs. Time Samples

FFT del segnale f(t)

Amplitude vs. Frequenza (Hz)
Time-Frequency Analysis: A Well-Known Example

(1.) There’s time for us.
Wavelet Transforms

\[ c(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \cdot \psi^* \left( \frac{t - b}{a} \right) dt \quad a \in R^+, \; b \in R \]

- Continuous WT, \( f(\tau) \) finite energy
- \( c(a, b) \) is a resemblance index between \( f(\tau) \) and \( \psi(\tau) \) located at a position \( b \) and scale \( a \) representing how closely correlated is the wavelet with a portion of the signal
- \( \psi(\tau) \) is localized in frequency and in time
Wavelet Analysis

Morlet wavelet function

\[ \psi(x) = C \cdot e^{-\frac{x^2}{2}} \cos(5x) \]
CWT Analysis

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Fourier Analysis

\[ f(n\tau) = \sin(2\pi f_1 n\tau) + \sin(2\pi f_2 n\tau) + \alpha[\delta_{n,n_1} + \delta_{n,n_2}] \]

- \( f_1 = 500 \text{Hz} \)
- \( f_2 = 1 \text{ KHz} \)
- \( \tau = 1/8000 \text{ s} \)
- \( \alpha = 1.5 \)
- \( n_1 = 250 \)
- \( n_2 = 282 \)
Wavelet Analysis

\[ \psi(t) = C e^{-\frac{t^2}{\alpha^2}} \left( e^{i\pi t} - e^{-\frac{\pi^2 \alpha^2}{4}} \right) \]
Radar/Sonar Applications

APPLICATIONS: airport Radar, metal detector, medical application (tissue imaging, velocity blood measurements)
DENOISING

- Problem: Radar/Sonar pulses detection and filtering in presence of strong noise and jamming signals

- Solution: using a thresholding procedure performed on coefficients resulting from a Wavelet Transform analysis
Experimental results

- System description
- Signal used to tune the filter

Received waveform in ideal case (f=8.4 Mhz, fc=1 Gs/s)

Amplitude (V)

Time (us)
Experimental results

- Signal corrupted by colored noise
- Signal spectrum

Received signal with colored noise: $F_s = 1$ Gs/sec

Received signal with colored noise
Experimental results

Filtered signal using the proposed algorithm

$RE=0.0396 @ PFD=-26.9082\text{dB}$
Denoising images (1)

• Algorithm Performance on a echografic image
Denoising Images (2)

- Enhancement of attenuation effects
Research topics: Ultrasounds

- Definition of algorithms
- Hardware implementations on FPGA board, on DSP, or Full Custom Design.
- Applications: Biomedical Imaging Enhancement, Tissues properties investigation…

“If you steal from one author it’s plagiarism, if you steal from many it’s research” W. Mizner
Data compression

• Fast Discrete algorithms

• WT renders sparse large classes of functions i.e. few noticeable coefficients many negligible

• Ex. Standard JPEG 2000
Research topics:
Music Signal Analysis

Wavelet Spectrogram

Midi Scores

Source: http://hil.t.u-tokyo.ac.jp
Research topics: Music Signal Analysis

- Definition of algorithms
- Hardware implementations on FPGA board, on DSP, or Full Custom Design.
- Applications: Music Information Retrieval, Sound Synthesis and Analysis…

“La musique est une mathématique mystérieuse dont les élément partecipent de l’infini” C. Debussy
Research topics: 
Device Simulation
Research topics: Device Simulation

- Definition of numerical algorithms
- Physical relevances analysis
- Computational Grid Automatic Design
- Software Engineering

“Entia non sunt multiplicanda praeter necessitatem”
Occam
Conclusions

- Wavelet Transform: a tool for time-frequency analysis
- Easy to implement: fast algorithms
- Well suited for many applications: such as non-stationary analysis or data compression
Wavelet Research Group

- Professors: Guido Masetti, Sergio Graffi, Nicolò Speciale.  
  (Sistemi Integrati per l’Analisi Spettrale LS)

- PhD Students: Emanuele Baravelli, Luca De Marchi, Matteo Montani, Nicola Testoni.

- Fellows: Salvatore Caporale, Francesco Franzè, Simona Maggio, Marco Messina, Alessandro Palladini.
Students Publications

