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AN INTRODUCTION TO WAVELET TRANSFORMS

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OUTLINE

- Time-Frequency Analysis
- Introduction on Wavelet Operators
- Examples of applications: Radar/Sonar
- Experimental results
- Conclusions

Fourier Analysis

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$f(t) = \frac{1}{2\Pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

- Fast Discrete Algorithm (FFT)
- FFT: a rotation in function space
- New basis functions sines and cosines
- Not localized in time

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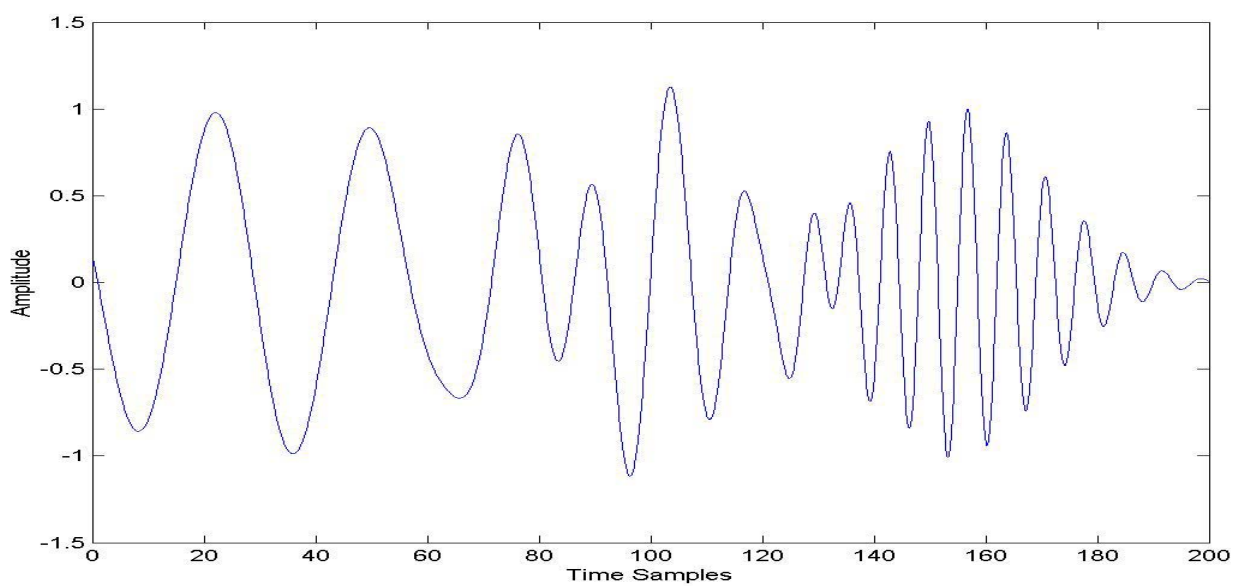
Signal Analysis

$$f(t) = f_1(t) + f_2(t) + f_3(t)$$

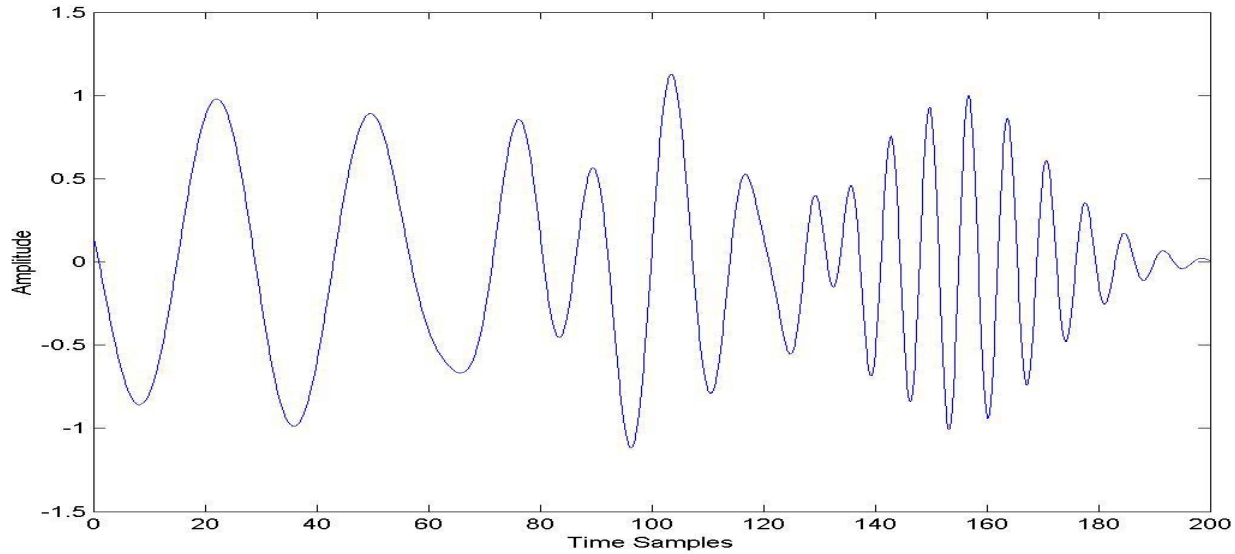
$$f_2(t) = \sin\left(2\pi \frac{t-100}{T_2}\right) e^{-\left(\frac{t-100}{1.8\,T_2}\right)^2} \qquad T_2 = 14$$

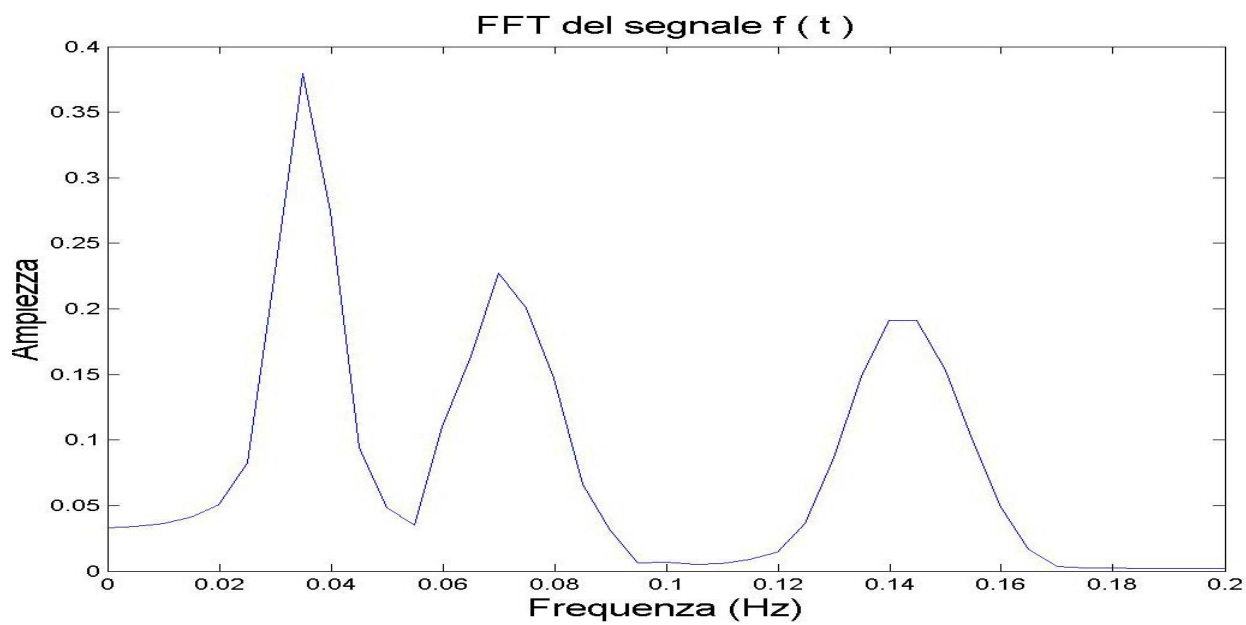
$$f_1(t) = \sin\left(2\pi \frac{t-30}{T_1}\right) e^{-\left(\frac{t-30}{2\,T_1}\right)^2} \qquad T_1 = 28$$

$$f_3(t) = \sin\left(2\pi \frac{t-155}{T_3}\right) e^{-\left(\frac{t-155}{3.2\,T_3}\right)^2} \qquad T_3 = 7$$

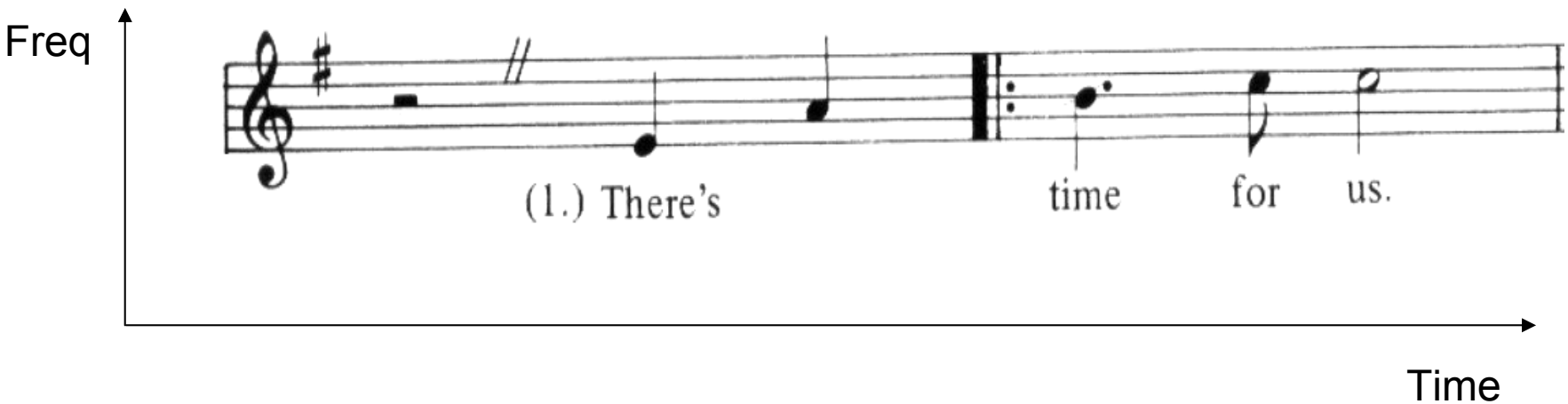


Fast Fourier Transform





Time-Frequency Analysis: A Well-Known Example

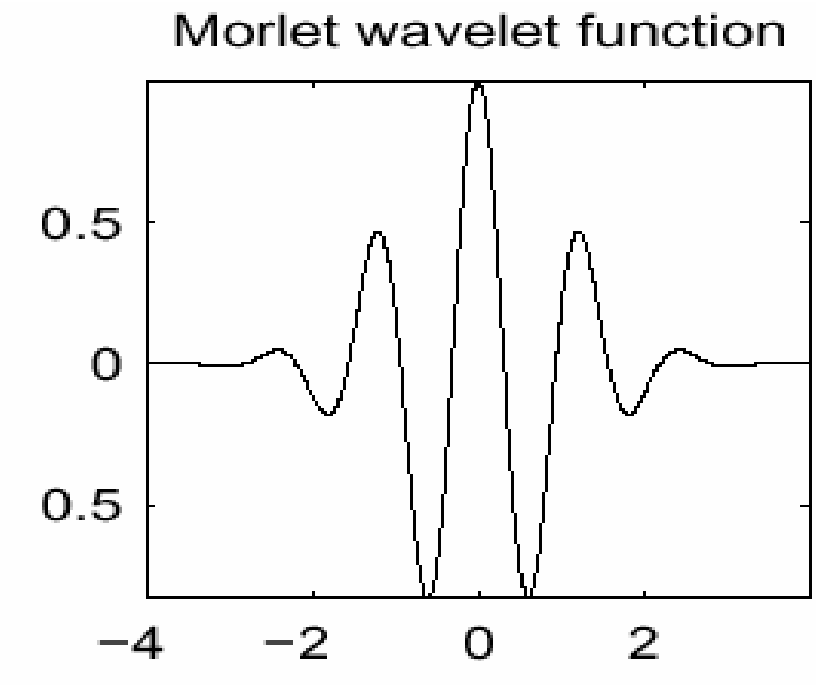


Wavelet Transforms

$$c(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \cdot \psi^* \left(\frac{t-b}{a} \right) dt \quad a \in R^+, \quad b \in R$$

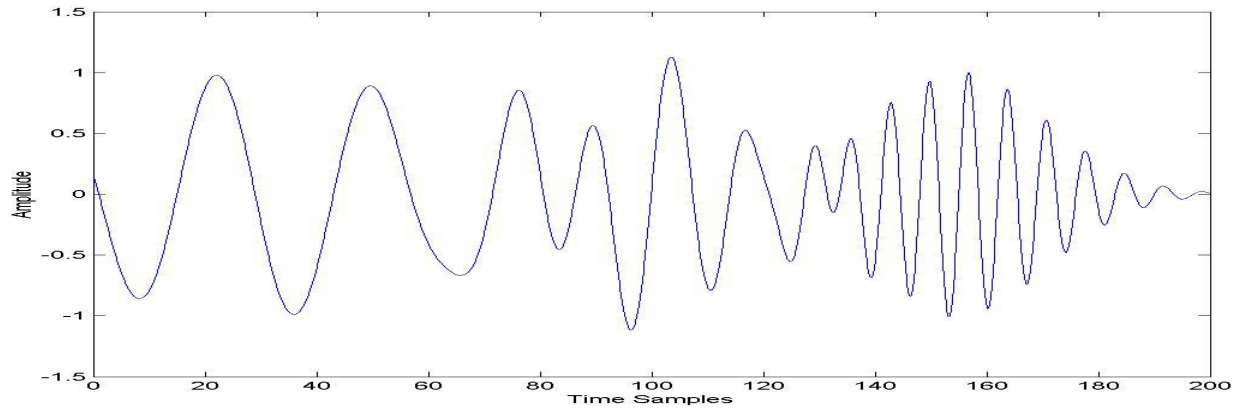
- Continuous WT, $f(\tau)$ finite energy
- c(**a**,**b**) is a resemblance index between $f(\tau)$ and $\psi(\tau)$ located at a position **b** and scale **a** representing how closely correlated is the wavelet with a portion of the signal
- $\psi(\tau)$ is localized in frequency and in time

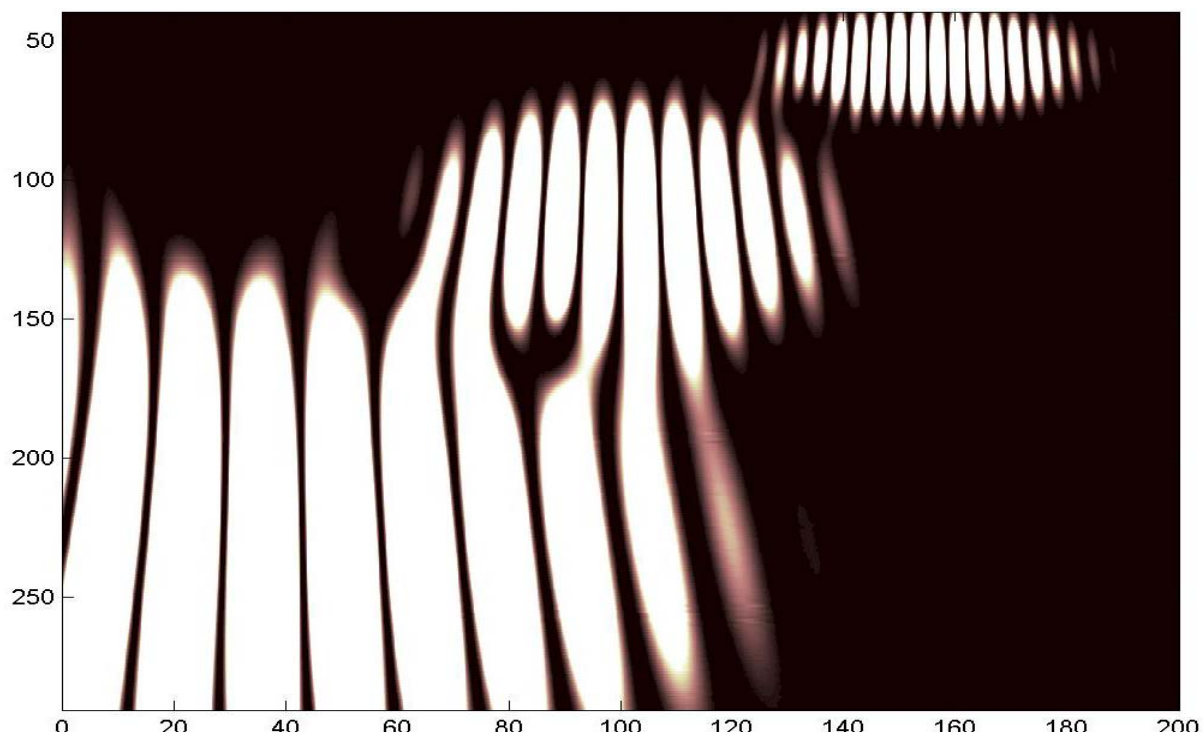
Wavelet Analysis



$$\psi(x) = C \cdot e^{-\frac{x^2}{2}} \cos(5x)$$

CWT Analysis





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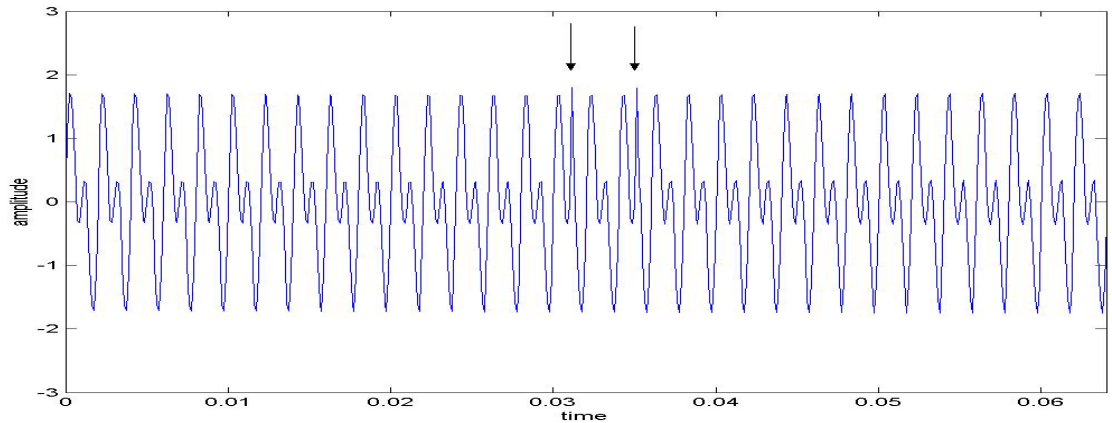
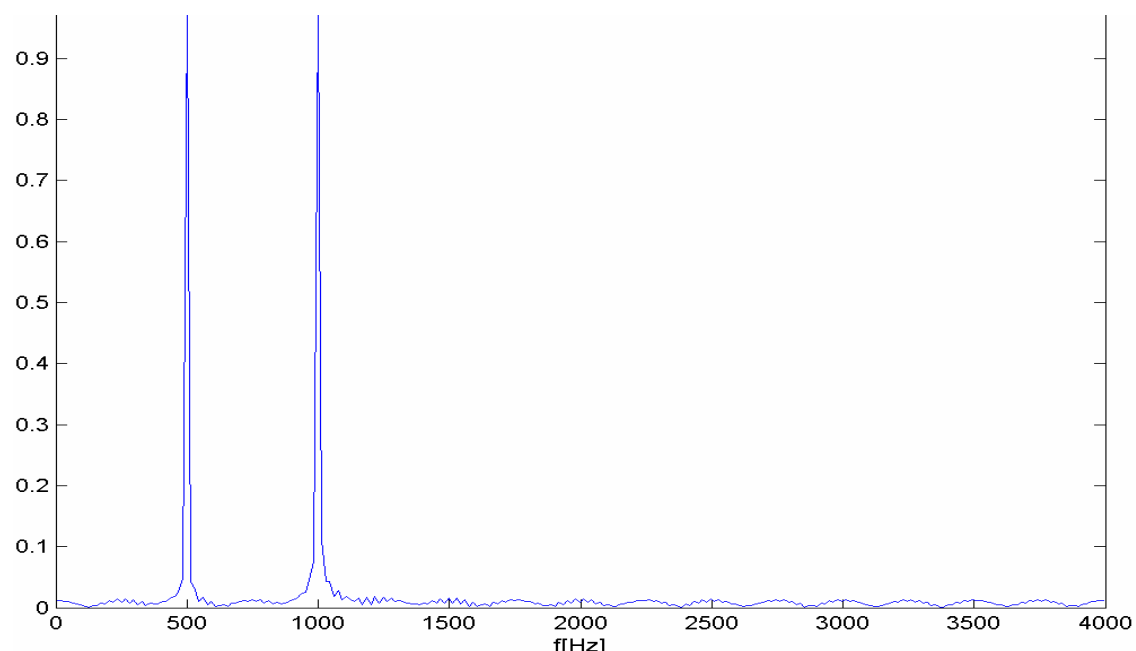
Fourier Analysis

$$f(n\tau) = \sin(2\pi f_1 n\tau) + \sin(2\pi f_2 n\tau) + \alpha[\delta_{n,n_1} + \delta_{n,n_2}]$$

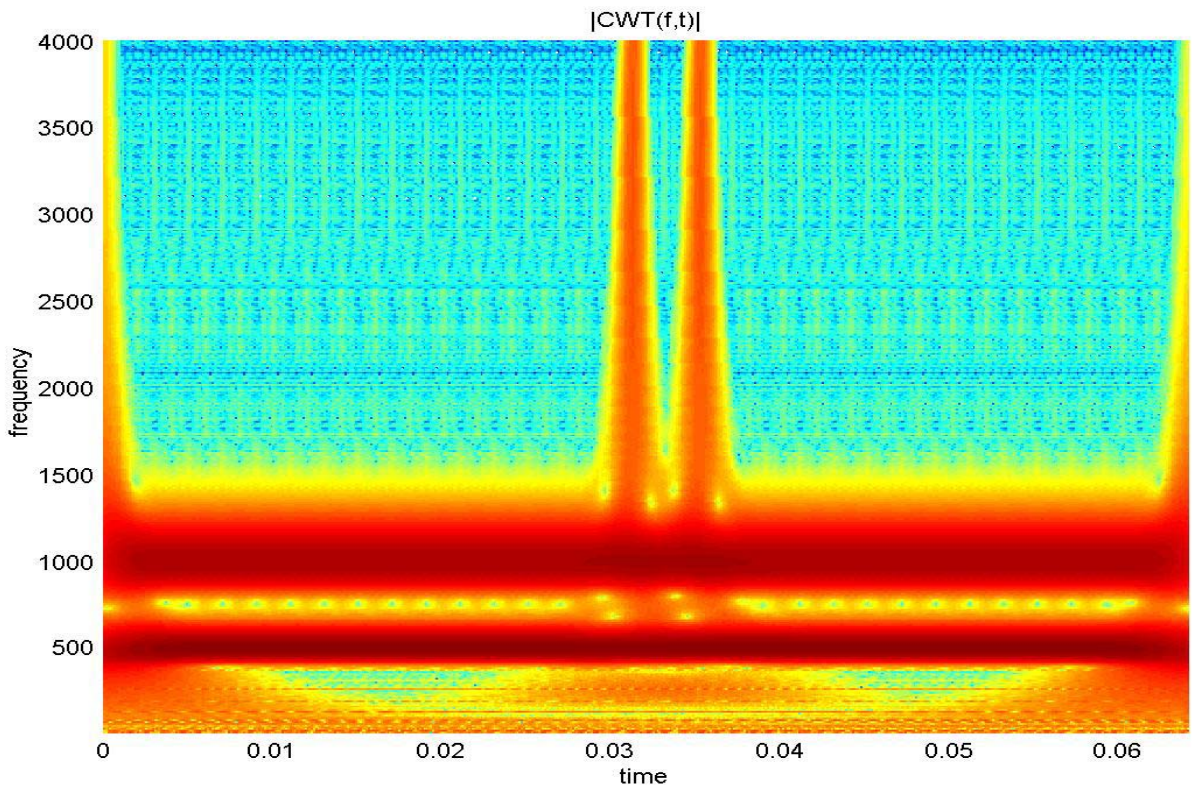


f_1= 500Hz
f_2=1 KHz
τ=1/8000 s
α=1.5
n_1=250
n_2=282

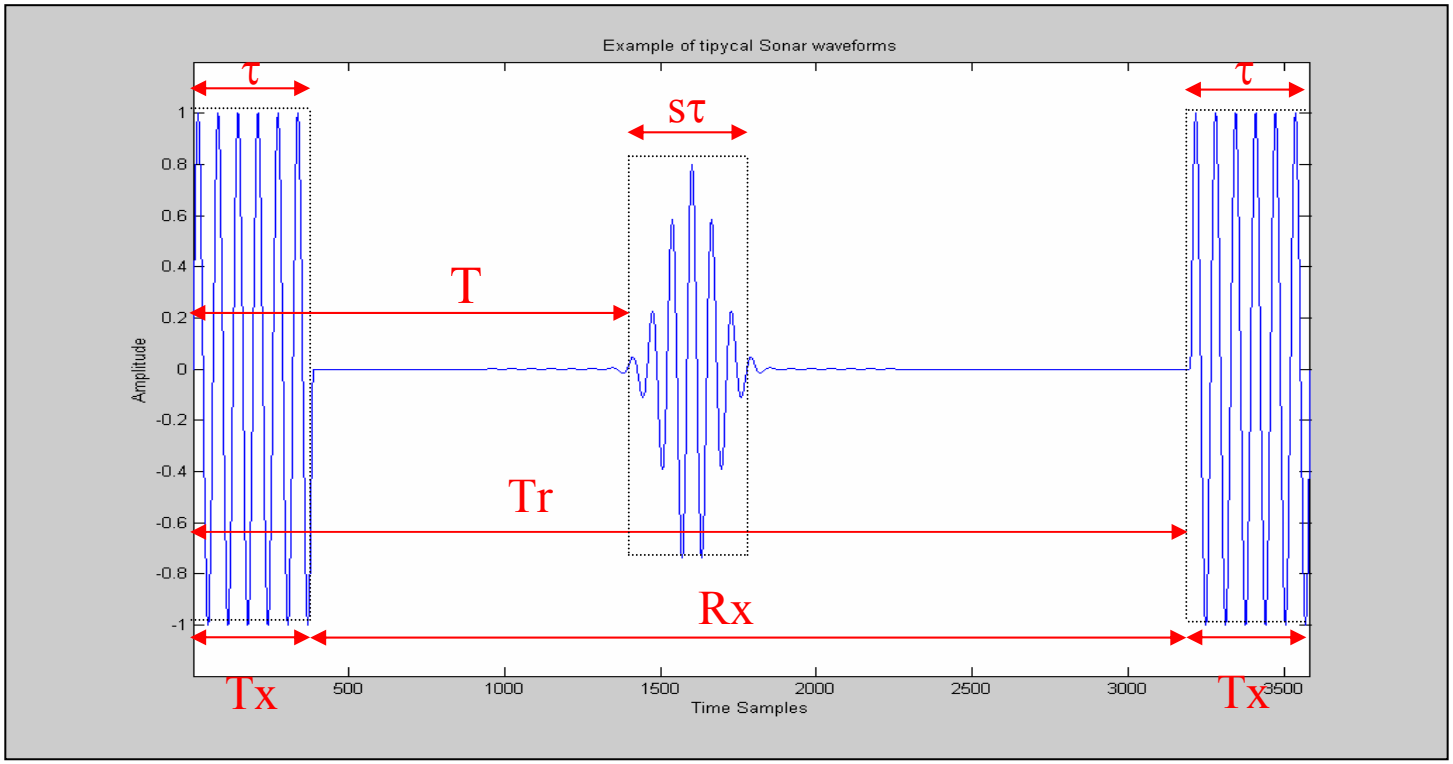


Wavelet Analysis

$$\psi(t) = Ce^{-t^2/\alpha^2}\left(e^{i\pi t} - e^{-\pi^2\alpha^2/4}\right)$$



Radar/Sonar Appplications



Radar Signal: fc=64Mhz, Tr=50us, τ=6us, fcarrier=1Mhz

APPLICATIONS: airport Radar, metal detector, medical application (tissue imaging, velocity blood measurements)

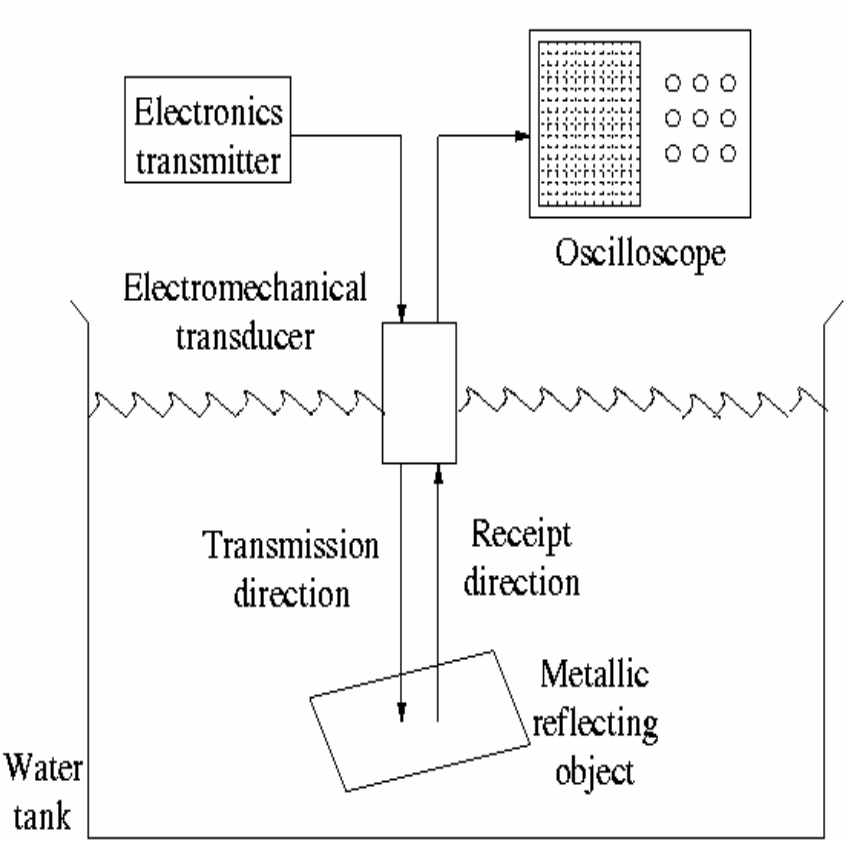
DENOISING

- Problem: Radar/Sonar pulses detection and filtering in presence of strong noise and jamming signals

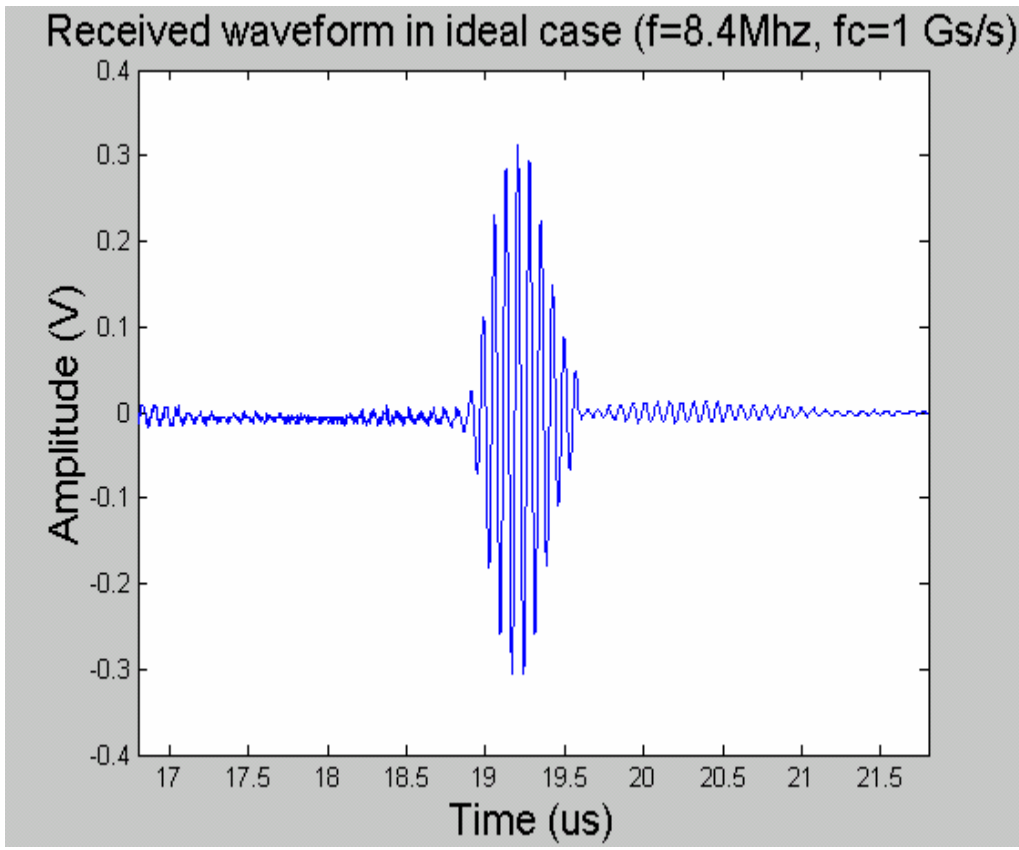
- Solution: using a thresholding procedure performed on coefficients resulting from a Wavelet Transform analysis

Experimental results

- System description

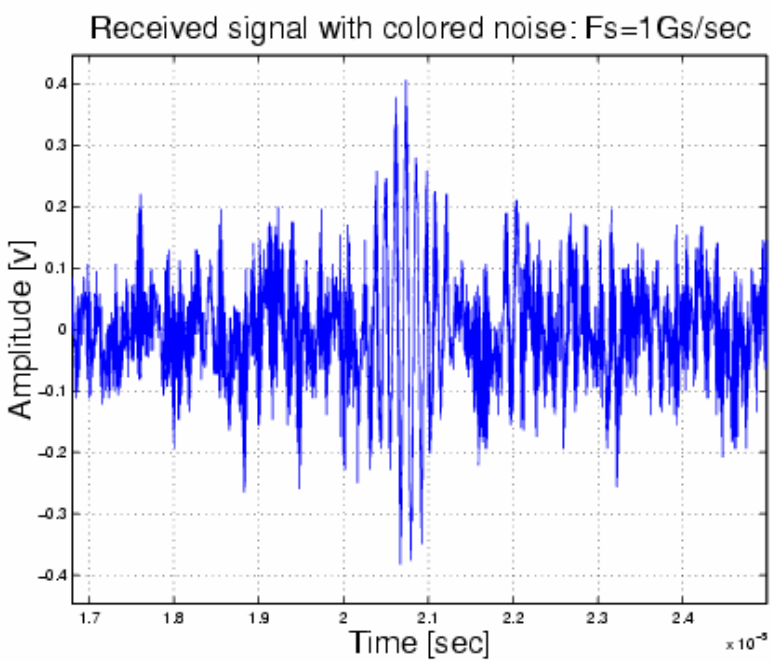


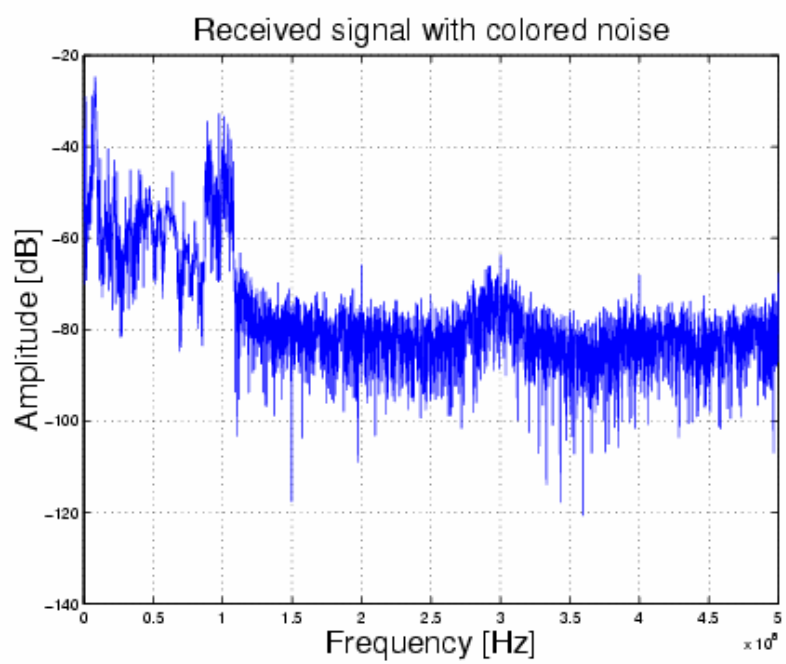
- Signal used to tune the filter



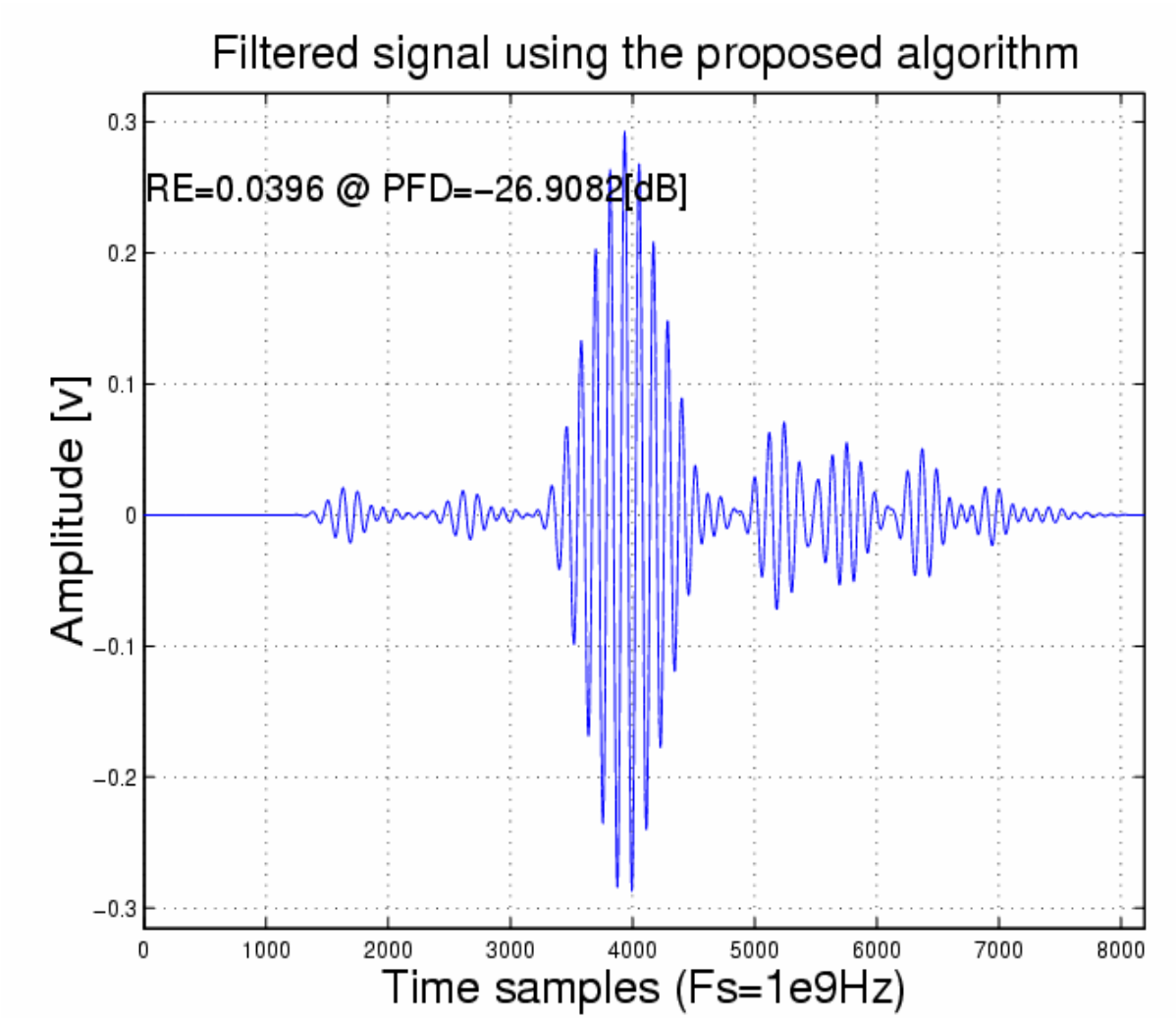
Experimental results

- Signal corrupted by colored noise
- Signal spectrum



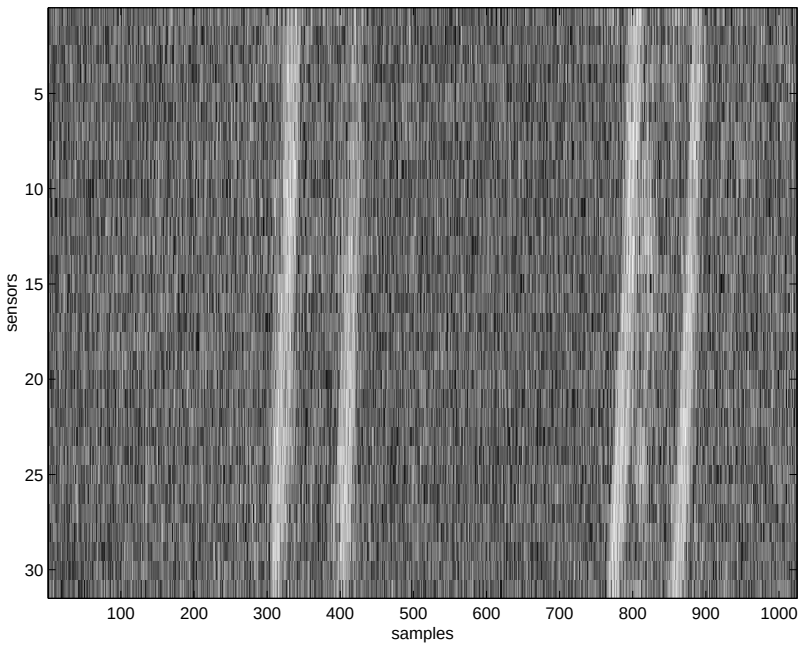


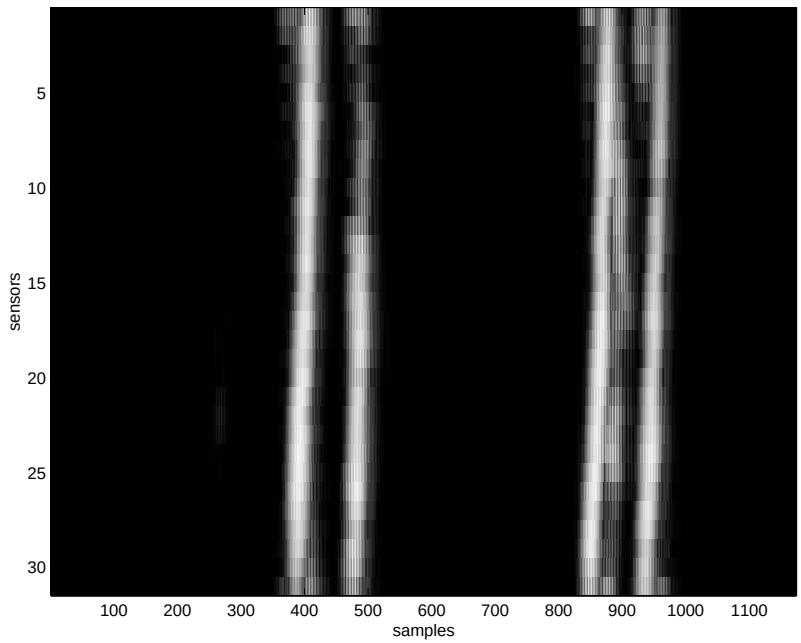
Experimental results



Denoising images (1)

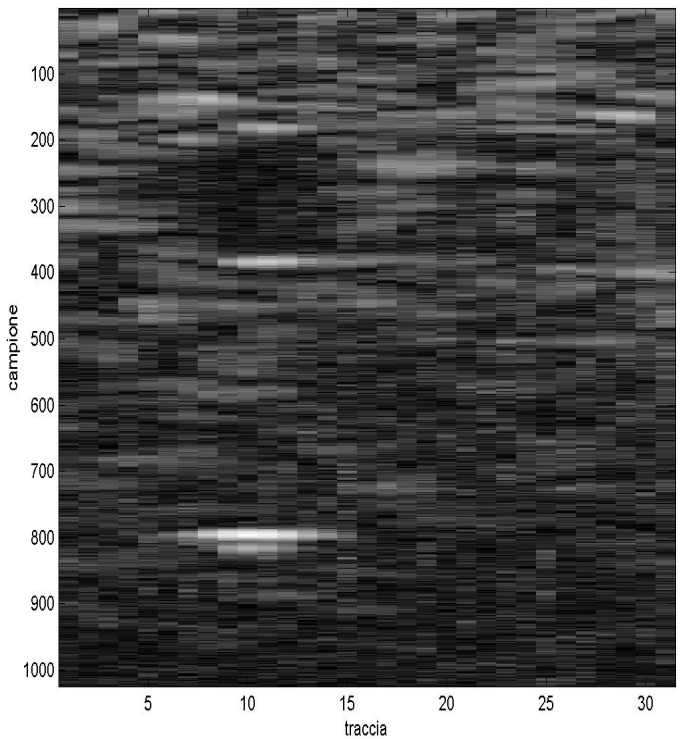
- Algorithm Performance on a echografic image

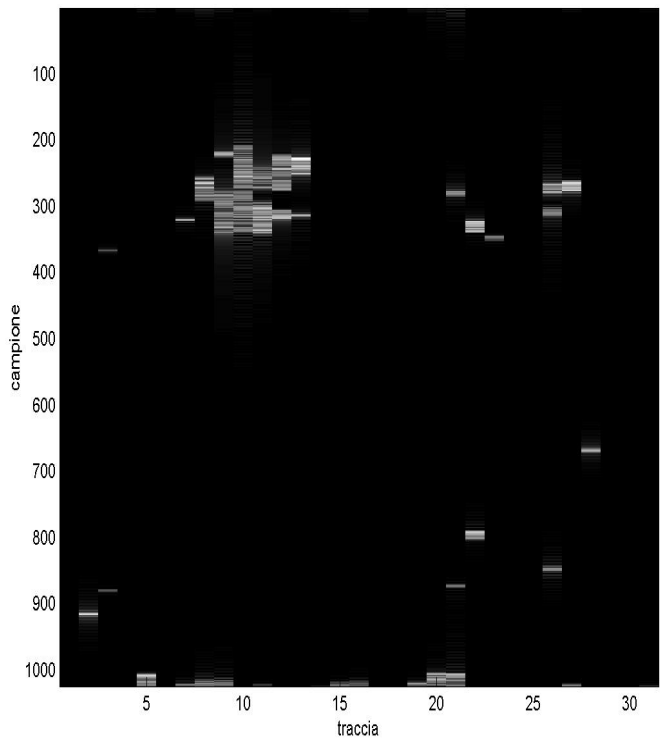




Denoising Images (2)

- Enhancement of attenuation effects





Research topics: Ultrasounds

- Definition of algorithms
- Hardware implementations on FPGA board, on DSP, or Full Custom Design.
- Applications: Biomedical Imaging Enhancement, Tissues properties investigation…

" If you steal from one author it's plagiarism, if you steal from many it's research" W.Mizner

Data compression

- Fast Discrete algorithms

- WT renders sparse large classes of functions
i.e. few noticeable coefficients many negligible

- Ex. Standard JPEG 2000

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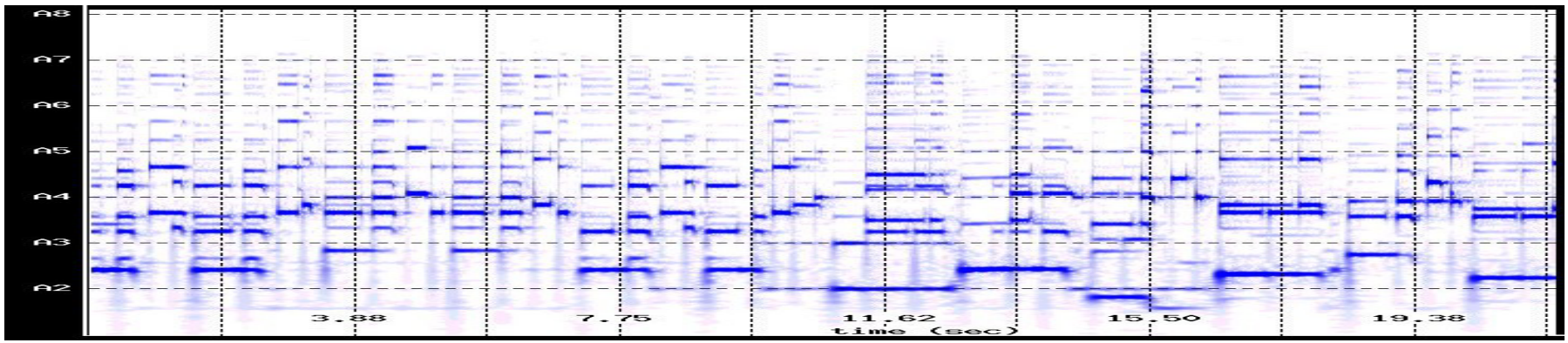
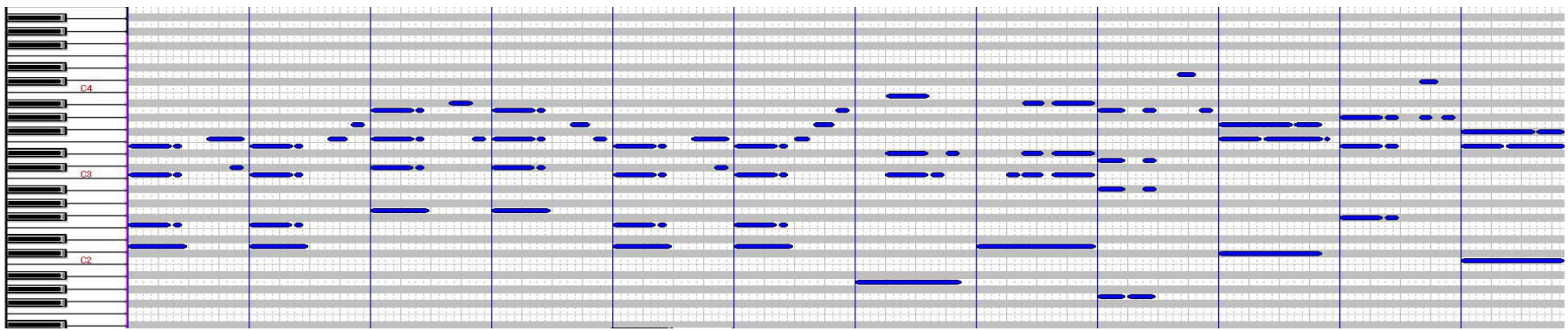
Research topics: Music Signal Analysis



Wavelet Spectrogram



Midi Scores

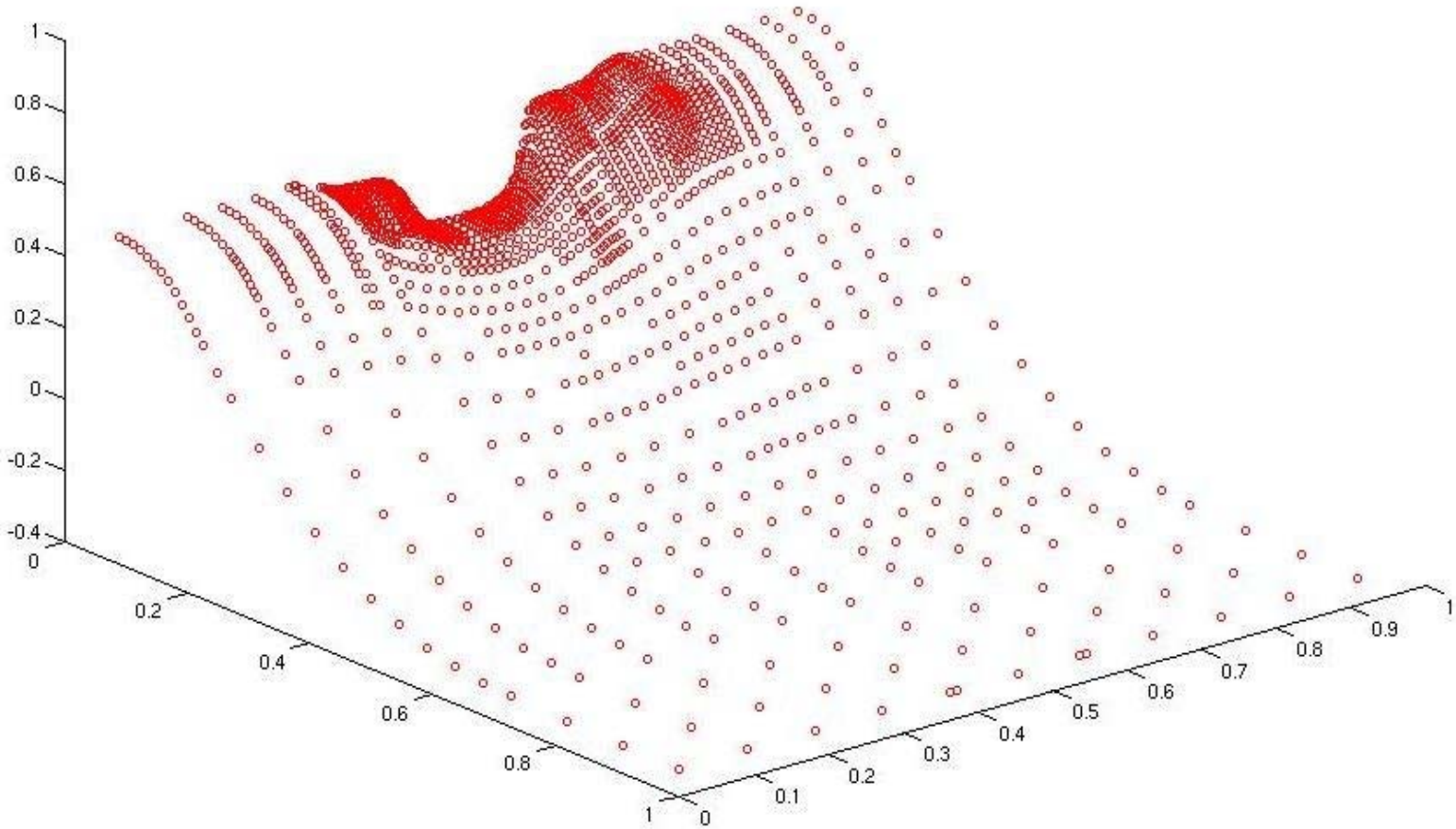
Source:http://hil.t.u-tokyo.ac.jp

Research topics: Music Signal Analysis

- Definition of algorithms
- Hardware implementations on FPGA board, on DSP, or Full Custom Design.
- Applications: Music Information Retrieval, Sound Synthesis and Analysis…

" La musique est une mathématique mystérieuse dont les élément partecipent de l'infini" C.Debussy

Research topics: Device Simulation



Research topics: Device Simulation

- Definition of numerical algorithms
- Physical relevances analysis
- Computational Grid Automatic Design
- Software Engineering

"Entia non sunt multiplicanda praeter necessitatem"

Occam

Conclusions

- Wavelet Transform: a tool for time -frequency analysis
- Easy to implement: fast algorithms
- Well suited for many applications: such as non-stationary analysis or data compression

Wavelet Research Group

- Professors: Guido Masetti, Sergio Graffi, Nicolò Speciale.
  (Sistemi Integrati per l'Analisi Spettrale LS)
- PhD Students: Emanuele Baravelli, Luca De Marchi, Matteo Montani, Nicola Testoni.
- Fellows: Salvatore Caporale, Francesco Franzè, Simona Maggio, Marco Messina, Alessandro Palladini.

Students Publications

- FPGA Implementation of QCWT Based Algorithm for filtering Low SNR Signals, A.Marcianesi, R.Padovani, N.Speciale, N.Testoni, G. Masetti, 2003.
- Wavelet-based Algorithms for Speckle Removal from B-Mode Images, S. Caporale, A. Palladini, L. De Marchi, N. Speciale, G. Masetti, 2004.
- Wavelet-based Deconvolution Algorithms Applied to Ultrasound Images, S. Maggio, N. Testoni, L. De Marchi, N. Speciale, G. Masetti, 2005.