Analog Filters for Telecommunications

Università' degli Studi di Bologna

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Introduction
Possible receiver architectures

• (Super)-Etherodyne

• Low-IF

• Direct conversion (Zero-IF)
Introduction

- Recently developed receiver uses Direct-conversion (Zero-IF) topologies
  Ex.: WLAN IEEE 802.11.a transceiver (Zhang@RF Micro Devices, ISSCC03)
Introduction

- WLAN IEEE 802.11.a transceiver (Bezhad@Broadcom, ISSCC03)

5th order Chebisev
Introduction

- WLAN IEEE 802.11.a transceiver (Vassiliou@Athena Semiconductors, ISSCC03)

DC offset, Synthesizer programming, Filter tuning

- Gm-C filter implementation
Introduction

- WCDMA receiver (Rogin@ETH-Zurich, ISSCC03)

- Active-RC filter implementation
Introduction

• Typically receiver filters are more challenging than transmitter filters

• Antenna GSM Input Spectrum

• Baseband GSM Input Spectrum
Introduction

Target of RX-filter

- rejecting out-of-band channels with high-linearity
  - to avoid in-band signal corruption of the intermodulation
- selecting in-band signals with low noise
Introduction
UMTS signal characteristics

• Adjacent channel

• Blockers
Baseband filters
UMTS receiver testing

• These tests are required for an UMTS receiver:
  ▪ **Adjacent channel test**
    ▪ In antenna
      • Signal = $-103\text{dBm}$ (14dB larger than the sensitivity)
      • Adjacent channel = $-52\text{dBm}$ with $\Delta f=5\text{MHz}$
    ▪ At BB Input (BB configured with the dc-gain 14dB lower than maximum value)
      • Signal = $-103\text{dBm} + 30\text{dB} = -73\text{dBm}$ (@ 1MHz)
      • Adjacent channel = $-52\text{dBm} + 30\text{dB} = -22\text{dBm}$ (@ 5MHz)
  ▪ **TEST:** The signal gain compression for AdjChann to be lower than 1dB
Baseband filters
UMTS receiver testing

- **In-Band Blockers Test**
  - In antenna
    - Signal = \(-114\)dBm
    - In-Band Blocker = \((-52\)dBm @ \(\Delta f=10\)MHz) or \((-44\)dBm @ \(\Delta f=15\)MHz)
  - At BB Input (BB configured with the maximum dc-gain)
    - Signal = \(-114\)dBm + 30dB = \(-84\)dBm (@ 1MHz)
    - In-Band Blocker = \(-56\)dBm + 30dB – 4dB (Mixer) = \(-22\)dBm (@ 10MHz)

- **TEST:** The signal gain compression due to In-Band Blocker has to be lower than 1dB
**Baseband filters**

**UMTS receiver testing**

- **Intermodulation Test**
  - In antenna
    - Signal = –114dBm
    - Two Interferers = (–46dBm @ Δf=10MHz) and (–46dBm @ Δf=20MHz)
  - At BB Input (BB configured with the maximum dc-gain)
    - Signal = –114dBm + 30dB = –84dBm (@ 1MHz)
    - Two Interferers = –46dBm+30dB–4dB(Mixer)=–22dBm (@10MHz&20MHz)

  - **TEST:** The in-band intermodulated component at the BB input has to be lower than the noise level (–99dBm (in antenna) + 30dB (RF-FE Gain) = –69dBm)
Baseband filters

Typical specifications

• Large linear range
  ▪ In-band linear range for avoiding tones intermodulation (OFDM)
  ▪ Out-of-band linear range for avoiding blockers intermodulation

  **Fully-differential solutions are always adopted**

• Accurate frequency response
  To select the signal bandwidth

• Frequency response programmability
  Multi-Mode / Multi-standard terminal (WLAN IEEE 802.11 a/b/g & UMTS)

• Low-noise in-band performance
• Low-power consumption (portable devices)
Analog Electronics Filters

Filter specification

• An electronic filters is an interconnection network of electrical components, which operates a modification of the frequency spectrum of an applied electrical signal

![Diagram of filter with frequency response](image)

• Different gain (amplitude and/or phase) for different signal frequencies
• The network is linear and time invariant
Filter Design Procedure

- Definition of the Filter specifications
- Design of the network that implements the specifications
- Sizing of the Component values
Filter Design Procedure

Filter specification

- The frequency response is usually defined by a mask, which specifies the range of allowed frequency responses
Filter Design Procedure

Filter specification

- The frequency range is divided into:
  - Pass-band
  - Stop-band

- Gain and Phase requirements
  - Gain requirements
    - Ripple in the pass band
    - Attenuation in the stop band
  - Phase requirements
    - Linear phase (constant group delay)
Filter Design Procedure

Filter Frequency Response

- Type of filters:
  - **Lowpass**, lowpass notch
  - Highpass, highpass notch
  - Bandpass, band-reject
  - Allpass (for phase equalizers)

- Filter specifications are met with linear networks with a transfer function of the form:

\[
H(s) = \frac{P_m(s)}{Q_n(s)}
\]

- \(P_m(s)\) and \(Q_n(s)\) are polynomial of order \(m\) and \(n\) respectively
Filter Design Procedure

Filter Frequency Response

\[ H(s) = \frac{P_m(s)}{Q_n(s)} \]

- The zeros of \( P_m(s) \) are the zeros of the transfer function
- The zeros of \( Q_n(s) \) are the poles of the transfer function

- Always \( n \geq m \)

\[ H(s) = \frac{a_0 + a_1 \cdot s + a_2 \cdot s^2 + a_3 \cdot s^3 + a_4 \cdot s^4 + \ldots}{b_0 + b_1 \cdot s + b_2 \cdot s^2 + b_3 \cdot s^3 + b_4 \cdot s^4 + \ldots} \]

\[ H(s) = \frac{(s + t_{z1}) \cdot (s + t_{z2}) \cdot \ldots \cdot (s^2 + s \cdot \omega_{z1} / Q_{z1} + \omega_{z1}^2) \cdot \ldots}{(s + t_{p1}) \cdot (s + t_{p2}) \cdot \ldots \cdot (s^2 + s \cdot \omega_{p1} / Q_{p1} + \omega_{p1}^2) \cdot \ldots} \]

- The number of poles gives the order of the filter
Analog filters

2nd order frequency response

- Lowpass Frequency Response

\[ H(s) = k \cdot \frac{\omega_z^2}{s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2} = \frac{a_0}{s^2 + b_1 \cdot s + b_0} \]

-40dB/dec

20 log Q''
Analog filters

2nd order frequency response

• Bandpass Frequency Response

\[ H(s) = k \cdot \frac{\omega_z \cdot s}{Q_z} = \frac{a_1 \cdot s}{s^2 + b_1 \cdot s + b_0} \]

\[ s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2 \]
Analog filters

2nd order frequency response

- Highpass Frequency Response

\[ H(s) = k \cdot \frac{s^2}{s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2} = \frac{a_2 \cdot s^2}{s^2 + b_1 \cdot s + b_0} \]
Analog filters

2nd order frequency response

- Notch Frequency Response

\[ H(s) = \frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p} \cdot s + \omega_p^2} = \frac{a_2 \cdot s^2 + a_0}{s^2 + b_1 \cdot s + b_0} \]
Analog filters

2nd order frequency response

- Frequency response vs. Q

(a) Bandpass

(b) Low-Pass

(c) High-Pass

(d) Notch

(e) All-Pass
Filter Design Procedure

Filter Frequency Response Approximations

- Lowpass Frequency Response

![Diagram showing possible approximations](image_url)
Filter Design Procedure

Filter Frequency Response Approximations

• Lowpass Frequency Response

• Any approximation implies trade-off in terms of
  Circuit complexity
  Power consumption
  Performance level
  linearity
  noise
  selectivity
Filter Design Procedure

Filter Frequency Response Approximations

- Bandpass Frequency Response

Possible Approximations

(a) Ideal mask
(b) Possible Approximation
(c) Possible Approximation
(d) Possible Approximation
(e) Possible Approximation
(f) Possible Approximation

A. Baschirotto, “Analog Filters for Telecommunications”
Filter Design Procedure
Filter Frequency Response Approximations

- Notch Frequency Response

(a) Notch Filter with Ideal Mask

(b) Ideal Notch Response

(c) Approximation (a) to (b)

(d) Approximation (b) to (c)

(e) Approximation (c) to (d)

(f) Approximation (d) to (e)

Possible Approximations
Filter Design Procedure

Filter Frequency Response Approximations

- Highpass Frequency Response

![Graphs of possible approximations for highpass frequency response](image)
Frequency Response Description

- Ideal lowpass Frequency Response

- Amplitude Response Limits for a Practical Low-Pass Filter

- Example of an Amplitude Response Curve Falling (set by $f_c$, $f_s$, $A_{\text{min}}$, and $A_{\text{max}}$)

- Another Amplitude Response Falling within the Desired Limits
Frequency Response Approximations

- Ideal lowpass frequency response

- Amplitude Response Limits for a Practical Low-Pass Filter

- Possible (most popular) approximations:
  - Butterworth (or Maximally Flat) Approximation
  - Chebyshev (or Equiripple) Approximation
  - Bessel (linear phase)
  - Elliptic (with transmission zeros)
Frequency Response Approximations

The Butterworth approximation

- The general equation for a Butterworth filter's amplitude response is:

\[ H(\omega) = \frac{1}{1 + \left( \frac{\omega}{\omega_0} \right)^{2n}} \]

- \( n \) is the order of the filter, and can be any positive whole number (1, 2, 3,...)
- \( \omega_0 \) is the -3dB frequency of the filter
Frequency Response Approximations

The Butterworth approximation

\[ H(\omega) = \frac{1}{1 + \left(\frac{\omega}{\omega_0}\right)^{2n}} \]

- Amplitude response curves for Butterworth filters of various filter orders

- The Butterworth (or maximally-flat response) exhibits a nearly flat passband with no ripple

- The rolloff is smooth and monotonic, with a low-pass or high-pass rolloff rate of 20 dB/decade (6 dB/octave) for every pole
Frequency Response Approximations

The Butterworth Approximation

In-band Amplitude Freq Response

Out-of-band Amplitude Freq Response

In-band Amplitude Phase Response
Frequency Response Approximations

The Butterworth approximation

Pole positions

<p>| TABLE 3.1 Poles of Lowpass Butterworth Filters |
|---|---|---|---|---|---|---|---|---|---|</p>
<table>
<thead>
<tr>
<th>n</th>
<th>Pole values</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.707107 ± 0.707107j</td>
</tr>
<tr>
<td>3</td>
<td>-1.0</td>
</tr>
<tr>
<td>4</td>
<td>-0.5 ± 0.866025j</td>
</tr>
<tr>
<td>5</td>
<td>-0.382683 ± 0.923880j</td>
</tr>
<tr>
<td>6</td>
<td>-0.258819 ± 0.965926j</td>
</tr>
<tr>
<td>7</td>
<td>-0.196090 ± 0.980785j</td>
</tr>
<tr>
<td>8</td>
<td>-0.555570 ± 0.831470j</td>
</tr>
<tr>
<td>9</td>
<td>-0.831470 ± 0.555570j</td>
</tr>
<tr>
<td>10</td>
<td>-0.980785 ± 0.195090j</td>
</tr>
</tbody>
</table>

Polynomial coefficients

<p>| TABLE 1(a). Butterworth Polynomials |
|---|---|---|---|---|---|---|---|---|---|</p>
<table>
<thead>
<tr>
<th>n</th>
<th>a₀</th>
<th>a₁</th>
<th>a₂</th>
<th>a₃</th>
<th>a₄</th>
<th>a₅</th>
<th>a₆</th>
<th>a₇</th>
<th>a₈</th>
<th>a₉</th>
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<td>1</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1.414</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2.000</td>
<td>2.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2.613</td>
<td>3.414</td>
<td>2.613</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>3.236</td>
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<td>5.236</td>
<td>3.236</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>5.759</td>
<td>16.592</td>
<td>31.163</td>
<td>41.966</td>
<td>41.966</td>
<td>31.163</td>
<td>16.592</td>
<td>5.759</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>6.392</td>
<td>20.432</td>
<td>42.802</td>
<td>64.882</td>
<td>74.233</td>
<td>64.882</td>
<td>42.802</td>
<td>20.432</td>
<td>6.392</td>
</tr>
</tbody>
</table>

| TABLE 1(b). Butterworth Quadratic Factors |
|---|---|
| n  | (s + 1) |
| 1  | (s² + 1.414² + 1) |
| 2  | (s + 1)(s² + 1) |
| 3  | (s + 1)(s² + 0.7654² + 1)(s² + 1.844² + 1) |
| 4  | (s + 1)(s² + 0.6180² + 1)(s² + 1.6180² + 1) |
| 5  | (s + 1)(s² + 0.5178² + 1)(s² + 1.414² + 1)(s² + 1.925² + 1) |
| 6  | (s + 1)(s² + 0.445² + 1)(s² + 1.247² + 1)(s² + 1.801² + 1) |
| 7  | (s + 1)(s² + 0.390² + 1)(s² + 1.111² + 1)(s² + 1.663² + 1)(s² + 1.982² + 1) |
| 8  | (s + 1)(s² + 0.347³ + 1)(s² + 1.000² + 1)(s² + 1.502² + 1)(s² + 1.879² + 1) |
| 9  | (s + 1)(s² + 0.212² + 1)(s² + 0.906² + 1)(s² + 1.414² + 1)(s² + 1.782² + 1)(s² + 1.975² + 1) |

• All the Tables&Plots are for a unitary cut-off frequency (ω₀=1rad/sec)
Frequency response approximations

The Butterworth approximation – Filter order choice

- The most stringent specification fixes the filter order
Frequency response approximations
The Butterworth approximation

• Step responses for Butterworth low-pass filters
• In each case $\omega_0 = 1$ and a unitary step amplitude is applied
• The step response is important when the filter is required to process a digital input signal, i.e. a bitstream
Frequency response approximations

The Chebyshev (or Equiripple) Approximation

- In the passband the ripple is fixed by the design
- The number of the flat point in the in-band frequency response corresponds to the filter order

Figure 4.2 Chebyshev response normalized to have pass-band end at $\omega = 1$ rad/s. Features are: (a) ripple limits, (b) pass band, (c) transition band, (d) stop band, and (e) intersection of response and lower ripple limit at $\omega = 1$. 
Frequency response approximations

The Chebyshev (or Equiripple) Approximation

- Different ripple values corresponds to different pole positions

<p>| TABLE 4.2 Pole Values for Lowpass Chebyshev Filters with 0.1-dB Pass-Band Ripple |
|---|---|</p>
<table>
<thead>
<tr>
<th>( n )</th>
<th>Pole values</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(-1.186178 \pm 1.380948j)</td>
</tr>
<tr>
<td>3</td>
<td>(-0.969406)</td>
</tr>
<tr>
<td></td>
<td>(-0.484703 \pm 1.206155j)</td>
</tr>
<tr>
<td>4</td>
<td>(-0.637730 \pm 0.465000j)</td>
</tr>
<tr>
<td></td>
<td>(-0.264156 \pm 1.122610j)</td>
</tr>
<tr>
<td>5</td>
<td>(-0.538914)</td>
</tr>
<tr>
<td></td>
<td>(-0.435991 \pm 0.687707j)</td>
</tr>
<tr>
<td></td>
<td>(-0.166534 \pm 1.080372j)</td>
</tr>
<tr>
<td>6</td>
<td>(-0.428041 \pm 0.283093j)</td>
</tr>
<tr>
<td></td>
<td>(-0.313348 \pm 0.773426j)</td>
</tr>
<tr>
<td></td>
<td>(-0.114693 \pm 1.056519j)</td>
</tr>
<tr>
<td>7</td>
<td>(-0.376778)</td>
</tr>
<tr>
<td></td>
<td>(-0.339465 \pm 0.463659j)</td>
</tr>
<tr>
<td></td>
<td>(-0.234917 \pm 0.835485j)</td>
</tr>
<tr>
<td></td>
<td>(-0.083841 \pm 1.041833j)</td>
</tr>
<tr>
<td>8</td>
<td>(-0.321650 \pm 0.205314j)</td>
</tr>
<tr>
<td></td>
<td>(-0.272682 \pm 0.584684j)</td>
</tr>
<tr>
<td></td>
<td>(-0.182200 \pm 0.875041j)</td>
</tr>
<tr>
<td></td>
<td>(-0.063980 \pm 1.032181j)</td>
</tr>
</tbody>
</table>

<p>| TABLE 4.3 Pole Values for Lowpass Chebyshev Filters with 0.5-dB Pass-Band Ripple |
|---|---|</p>
<table>
<thead>
<tr>
<th>( n )</th>
<th>Pole values</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(-0.712812 \pm 1.00402j)</td>
</tr>
<tr>
<td>3</td>
<td>(-0.626457)</td>
</tr>
<tr>
<td></td>
<td>(-0.313228 \pm 1.021928j)</td>
</tr>
<tr>
<td>4</td>
<td>(-0.423340 \pm 0.420946j)</td>
</tr>
<tr>
<td></td>
<td>(-0.175353 \pm 1.016253j)</td>
</tr>
<tr>
<td>5</td>
<td>(-0.362320)</td>
</tr>
<tr>
<td></td>
<td>(-0.293123 \pm 0.625177j)</td>
</tr>
<tr>
<td></td>
<td>(-0.111963 \pm 1.011557j)</td>
</tr>
<tr>
<td>6</td>
<td>(-0.289794 \pm 0.270216j)</td>
</tr>
<tr>
<td></td>
<td>(-0.212144 \pm 0.738245j)</td>
</tr>
<tr>
<td></td>
<td>(-0.077650 \pm 1.008461j)</td>
</tr>
<tr>
<td>7</td>
<td>(-0.256170)</td>
</tr>
<tr>
<td></td>
<td>(-0.230801 \pm 0.447894j)</td>
</tr>
<tr>
<td></td>
<td>(-0.159719 \pm 0.807077j)</td>
</tr>
<tr>
<td></td>
<td>(-0.057003 \pm 1.006409j)</td>
</tr>
<tr>
<td>8</td>
<td>(-0.219293 \pm 0.199907j)</td>
</tr>
<tr>
<td></td>
<td>(-0.185908 \pm 0.569288j)</td>
</tr>
<tr>
<td></td>
<td>(-0.124219 \pm 0.852000j)</td>
</tr>
<tr>
<td></td>
<td>(-0.043620 \pm 1.005002j)</td>
</tr>
</tbody>
</table>
Frequency response approximations

The Chebyshev Approximation: Even / Odd order Frequency Response

- Even order filter frequency response
- Odd order filter frequency response
Frequency response approximations

The Chebyshev Approximation: Out-of-Band Frequency Response

Figure 4.7  Stop-band magnitude response of lowpass Chebyshev filters with 0.5 dB ripple.

Figure 4.8  Phase response of lowpass Chebyshev filters with 0.5-dB pass-band ripple.

Figure 4.12  Step response of lowpass Chebyshev filters with 0.5-dB pass-band ripple.
Frequency response approximations

The Bessel approximation

- Optimum for step response
Frequency response approximations

The Bessel approximation

4\textsuperscript{th}-order Butterworth lowpass filter

- The ``ringing'' in the response shows that the nonlinear phase shift distorts the filtered wave shape

4th-order Bessel low-pass filter

- No ringing in the response.
- Rounding of the corners due to the reduction of high frequency components
- The response is a relatively undistorted version of the input square wave
Frequency response approximations

The Bessel approximation

Figure 6.1 Pass-band magnitude response of lowpass Bessel filters.

Figure 6.2 Stop-band magnitude response of lowpass Bessel filters.
Frequency Response Approximations

The Bessel approximation

Group delay:

\[ \tau_g(\omega) = - \frac{d\varphi(\omega)}{d\omega} \]

Figure 6.3 Group-delay response of lowpass Bessel filters.
Frequency Response Approximations

The Bessel approximation

<table>
<thead>
<tr>
<th>TABLE 6.2</th>
<th>Poles of Bessel Filter Normalized to Have Unit Delay at $\omega = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>Pole values</td>
</tr>
<tr>
<td>2</td>
<td>$-1.5 \pm 0.8660j$</td>
</tr>
<tr>
<td>3</td>
<td>$-2.3222$</td>
</tr>
<tr>
<td></td>
<td>$-1.8390 \pm 1.7543j$</td>
</tr>
<tr>
<td>4</td>
<td>$-2.1039 \pm 2.6575j$</td>
</tr>
<tr>
<td></td>
<td>$-2.8961 \pm 0.8672j$</td>
</tr>
<tr>
<td>5</td>
<td>$-3.6467$</td>
</tr>
<tr>
<td></td>
<td>$-2.3247 \pm 3.5710j$</td>
</tr>
<tr>
<td></td>
<td>$-3.3620 \pm 1.7427j$</td>
</tr>
<tr>
<td>6</td>
<td>$-2.5158 \pm 4.4927j$</td>
</tr>
<tr>
<td></td>
<td>$-3.7357 \pm 2.6263j$</td>
</tr>
<tr>
<td></td>
<td>$-4.2484 \pm 0.8675j$</td>
</tr>
<tr>
<td>7</td>
<td>$-4.9716$</td>
</tr>
<tr>
<td></td>
<td>$-2.6857 \pm 5.4206j$</td>
</tr>
<tr>
<td></td>
<td>$-4.0701 \pm 3.5173j$</td>
</tr>
<tr>
<td></td>
<td>$-4.7584 \pm 1.7393j$</td>
</tr>
<tr>
<td>8</td>
<td>$-5.2049 \pm 2.6162j$</td>
</tr>
<tr>
<td></td>
<td>$-4.3683 \pm 4.4146j$</td>
</tr>
<tr>
<td></td>
<td>$-2.8388 \pm 6.3540j$</td>
</tr>
<tr>
<td></td>
<td>$-5.5878 \pm 0.8676j$</td>
</tr>
</tbody>
</table>
Frequency Response Approximations

Different approximations comparison

- Magnitude responses of the Butterworth third-order lowpass filter and the corresponding Chebyshev (1 dB ripple) and Bessel filters.
Frequency Response Approximations

Different approximations comparison: Summary

• Butterworth Response
  😊 Maximally flat magnitude response in the pass-band
  😊 Its pulse response is better than Chebyshev
  😊 Its rate of attenuation is better than that of Bessel
  😞 Some overshoot and ringing is exhibited in step response

• Chebyshev Response
  😊 Better attenuation beyond the pass-band than Butterworth
  😞 Ripple in pass-band may be objectionable
  😞 A considerable ringing in step response

• Bessel Response
  😊 The best step response: very little overshoot or ringing
  😞 A slower rate of attenuation beyond the pass-band than Butterworth
Frequency Response Approximations

The elliptic approximation

- Example of a elliptic low-pass amplitude response
- This particular filter is 4\textsuperscript{th}-order with $A_{\text{max}}=0.5$ dB and $f_s/f_c=2$
Designing a filter using the Tables

Summary

- The above tables refer to the case of a prototype filter with characteristic frequency equal to $\omega_C=1\text{rad/sec}$

- They can be used (with proper transformations) to
  - Synthesize low-pass filter with arbitrary $\omega_C$
  - Synthesize band-pass filter with arbitrary center frequency and passband
  - Synthesize high-pass filter with arbitrary $\omega_C$

- Typically this is done automatically by everywhere available software:
  - MATLAB: filtdemo
  - etc......
Designing a filter using the Tables

Lowpass design example

• A linear phase (Bessel) 2\textsuperscript{nd} order filter with $\omega_C = 200$ rad/sec is designed

• Bessel prototype poles

$$\omega_C = 1 \text{ rad/sec}$$

$$p_{1,2} = -1.5 \pm 0.866\times j$$

• Modified Bessel poles

$$\omega_C = 200 \text{ rad/sec}$$

$$p_{1,2} = ( -1.5 \pm 0.866\times j ) \times 200$$
MATLAB Functions for Analog Filter Design

• Transfer function plot

FREQS Laplace-transform (s-domain) frequency response.
H = FREQS(B,A,W) returns the complex frequency response vector H
of the filter B/A:

\[ H(s) = \frac{\text{numerator terms}}{\text{denominator terms}} \]

\[ H(s) = \frac{b(1)s^{nb-1} + b(2)s^{nb-2} + \ldots + b(nb)}{a(1)s^{na-1} + a(2)s^{na-2} + \ldots + a(na)} \]

given the numerator and denominator coefficients in vectors B and A.
The frequency response is evaluated at the points specified in
vector W (in rad/s). The magnitude and phase can be graphed by
calling FREQS(B,A,W) with no output arguments.

[H,W] = FREQS(B,A) automatically picks a set of 200 frequencies W on
which the frequency response is computed. FREQS(B,A,N) picks N
frequencies.

See also LOGSPACE, POLYVAL, INVFREQS, and FREQZ.
MATLAB Functions for Analog Filter Design

• Butterworth filter order selection

BUTTORD Butterworth filter order selection.

[N, Wn] = BUTTORD(Wp, Ws, Rp, Rs) returns the order N of the lowest order digital Butterworth filter that loses no more than Rp dB in the passband and has at least Rs dB of attenuation in the stopband. Wp and Ws are the passband and stopband edge frequencies, normalized from 0 to 1 (where 1 corresponds to pi radians/sample). For example,

Lowpass: \( Wp = 0.1, \quad Ws = 0.2 \)

Highpass: \( Wp = 0.2, \quad Ws = 0.1 \)

Bandpass: \( Wp = [0.2, 0.7], \quad Ws = [0.1, 0.8] \)

Bandstop: \( Wp = [0.1, 0.8], \quad Ws = [0.2, 0.7] \)

BUTTORD also returns Wn, the Butterworth natural frequency (or, the "3 dB frequency") to use with BUTTER to achieve the specifications.

[N, Wn] = BUTTORD(Wp, Ws, Rp, Rs, 's') does the computation for an analog filter, in which case Wp and Ws are in radians/second.

When Rp is chosen as 3 dB, the Wn in BUTTER is equal to Wp in BUTTORD.

See also BUTTER, CHEB1ORD, CHEB2ORD, ELLIPORD.
MATLAB Functions for Analog Filter Design

• Butterworth Transfer function calculation

BUTTER Butterworth digital and analog filter design.

[B,A] = BUTTER(N,Wn) designs an Nth order lowpass digital Butterworth filter and returns the filter coefficients in length N+1 vectors B (numerator) and A (denominator). The coefficients are listed in descending powers of z. The cutoff frequency Wn must be 0.0 < Wn < 1.0, with 1.0 corresponding to half the sample rate.

If Wn is a two-element vector, Wn = [W1 W2], BUTTER returns an order 2N bandpass filter with passband W1 < W < W2.

[B,A] = BUTTER(N,Wn,'high') designs a highpass filter.

[B,A] = BUTTER(N,Wn,'stop') is a bandstop filter if Wn = [W1 W2].

When used with three left-hand arguments, as in

[Z,P,K] = BUTTER(...), the zeros and poles are returned in length N column vectors Z and P, and the gain in scalar K.

When used with four left-hand arguments, as in

[A,B,C,D] = BUTTER(...), state-space matrices are returned.

BUTTER(N,Wn,'s'), BUTTER(N,Wn,'high','s') and BUTTER(N,Wn,'stop','s') design analog Butterworth filters. In this case, Wn is in [rad/s] and it can be greater than 1.0.

See also BUTTORD, BESSELF, CHEBY1, CHEBY2, ELLIP, FREQZ, FILTER.
MATLAB Functions for Analog Filter Design

Design example: Lowpass filter with fo=30kHz with 2dB and -30dB stopband at 100kHz

STEP 1 – Filter order evaluation

>> [N, Wn] = buttord(30e3*2*pi, 100e3*2*pi, 2, 30, 's')
N =
    4
Wn =
    2.6499e+05

STEP 2 – Transfer function calculation

>> [num,den]=butter(N,Wn,'s')
num =
    1.0e+21 *
    0         0         0         0    4.9310
    0         0         0         0    0.0000
    0         0         0         0    0.0000
    0         0         0         0    0.0000
    4.9310

den =
    1.0e+21 *
    0.0000    0.0000    0.0000    0.0000    4.9310
    0.0000    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000    0.0000
    4.9310
>> den'
ans =
    1.000000000000000e+00
    6.924597294806722e+05
    2.397502384762228e+11
    4.862536635311685e+16
    4.931019606075012e+21

STEP 3 – Filter transfer function plot

>> wf=2*pi*(1e3:1e3:200e3);
>> freqs(num,den,wf)
>> mod=20*log10(abs(freqs(num,den,wf)));
>> plot(wf/(2*pi),mod)
High-order filters

• There are several ways to realize high order (> 2) filters:

• Cascade realization

• Multiple loop feedback
High order filters
Cascade realization

- It consists in the cascade connection of isolated 1st order & biquad sections
- $H_i(s)$ is the biquad transfer function

$$H(s) = \frac{V_o}{V_i} = \prod_{i=1}^{N} H_i(s)$$

- For getting isolation it must be:

  $$R_s << Z_{\text{in},1}$$
  $$Z_{\text{out},1} << Z_{\text{in},i+1}$$
  $$Z_{\text{out},N} << R_L$$
High order filters

Cascade realization

\[ H(s) = \frac{(s + t_{z1}) \cdot (s + t_{z2}) \cdot \ldots \cdot (s^2 + s \cdot \omega_{z1} / Q_{z1} + \omega_{z1}^2) \cdot \ldots}{(s + t_{p1}) \cdot (s + t_{p2}) \cdot \ldots \cdot (s^2 + s \cdot \omega_{p1} / Q_{p1} + \omega_{p1}^2) \cdot \ldots} \]

\[ H(s) = \frac{(s + t_{z1}) \cdot (s + t_{z2})}{(s + t_{p1}) \cdot (s + t_{p2})} \ldots \frac{(s^2 + s \cdot \omega_{z1} / Q_{z1} + \omega_{z1}^2)}{(s^2 + s \cdot \omega_{p1} / Q_{p1} + \omega_{p1}^2)} \ldots \]

- Design problem: Group together pole and zero pairs (pole-zero pairing)
  - It affects the dynamic range and the sensitivity.
  - There are not general and consistent rules for the pole-zero pairing in order to minimize the sensitivity.

- Two opposite approaches are suggested in the literature:
  - Pair the high-Q poles with far away zeros
  - Pair the high-Q poles with closed zeros

- The only solution is to try different pairing and to compare them with Monte Carlo analysis

- For order > 6 the cascade design is inherently more sensitive to component variation than multiple-loop feedback realizations (Is it true for the phase ??)
High order filters

Multiple loop feedbacks

The previous 1\textsuperscript{st} and 2\textsuperscript{nd} order cells can be used to build up high order filters

• Several topologies:
  
  • Follow the leader feedback (FLF)
  
  • Inverse follow the leader (IFLF)
  
  • Generalize follow the leader feedback (GFLF)
  
  • Primary resonator block (PRB)
  
  • Leapfrog feedback (LF)
  
  • Modified leapfrog feedback (MLF)
  
  • Coupled biquad (CB)
  
  • Minimum sensitivity feedback (MSF)
SC Ladder filters

- Passive LC ladder filters exhibit in the passband very low sensitivities to component variations.
- From LC passive ladders, the same networking philosophy has been reproduced in other filtering techniques.
- In 60's the active ladder filters were made of resistors, capacitors and op amps.

- Typical all-pole low-pass $RLC$ network (5th order), doubly and equally terminated for best sensitivity performance.
  
  ![RLC network diagram]

- The active realization will implement the same current-voltage relationships of capacitors and inductors of the passive networks.
- These relationships are of integral nature.
- $\Rightarrow$ They will be naturally implemented by using first order integrators.
Leapfrog Topology

Design procedure

- Simulate passive ladder networks, via signal flow graph

\[ I_0 = \frac{1}{R_S} \cdot (V_{in} - V_1) \]
\[ V_1 = \frac{1}{s \cdot C_1} \cdot (I_0 - I_2) \]
\[ I_2 = \frac{1}{s \cdot L_2} \cdot (V_1 - V_3) \]
\[ V_3 = \frac{1}{s \cdot C_3} \cdot (I_2 - I_4) \]
\[ I_4 = \frac{1}{s \cdot L_4} \cdot (V_3 - V_5) \]
\[ V_5 = \frac{1}{s \cdot C_5} \cdot (I_4 - I_6) \]
\[ I_6 = \frac{1}{R_S} \cdot V_5 \]
\[ V_{out} = V_5 \]

- Multiply each current by an arbitrary resistance

\[ V_0 = R \cdot I_0; \quad V_2 = R \cdot I_2; \quad V_4 = R \cdot I_4; \quad V_6 = R \cdot I_6 \]
Leapfrog Topology

Design procedure

\[ V_0 = R \cdot I_0; \quad V_2 = R \cdot I_2; \quad V_4 = R \cdot I_4; \quad V_6 = R \cdot I_6 \]

The obtained set of equation is

\[ V_{\text{in}} = \frac{R}{R_S} \cdot (V_{\text{in}} - V_1) \]
\[ V_1 = \frac{1}{s \cdot R \cdot C_1} \cdot (V_0 - V_2) \]
\[ V_2 = \frac{R}{s \cdot L_2} \cdot (V_1 - V_3) \]
\[ V_3 = \frac{1}{s \cdot R \cdot C_3} \cdot (V_2 - V_4) \]
\[ V_4 = \frac{R}{s \cdot L_4} \cdot (V_3 - V_5) \]
\[ V_5 = \frac{1}{s \cdot R \cdot C_5} \cdot (V_4 - V_6) \]
\[ V_6 = \frac{R}{R_S} V_5 \]
\[ V_{\text{out}} = V_5 \]
Leapfrog Topology

Design procedure

• The original set of equations and the modified one can be represented with the signal flow graphs:

\[
V_0 = \frac{R}{R_S} \cdot (V_{in} - V_1)
\]

\[
V_1 = \frac{1}{s \cdot R \cdot C_1} \cdot (V_0 - V_2)
\]

\[
V_2 = \frac{R}{s \cdot L_2} \cdot (V_1 - V_3)
\]

\[
V_3 = \frac{1}{s \cdot R \cdot C_3} \cdot (V_2 - V_4)
\]

\[
V_4 = \frac{R}{s \cdot L_4} \cdot (V_3 - V_5)
\]

\[
V_5 = \frac{1}{s \cdot R \cdot C_5} \cdot (V_4 - V_6)
\]

\[
V_6 = \frac{R}{R_S} V_5
\]

\[
V_{out} = V_5
\]
Leapfrog Topology
Design procedure

• Scaling of the flow graph:

Variable transformation

Scaling by constants

• The scaling of the flow graph is used in order to obtain realizable active implementations
Leapfrog Topology

Design procedure

- The electrical network is obtained by replacing the integrators with a circuit implementation.

- Interconnection of second order loops ($R \approx R_s \approx R_6$)
Leapfrog Topology

Design procedure

- The scaling is also useful for dynamic range optimization:

- $V_1$ is too small $V_2$ is too high

$$V_1 \rightarrow K_1 \cdot V_1$$
$$V_2 \rightarrow V_1 / K_2$$

- **Result:** all the op-amp saturate (at different frequency) with the same input level

- Perform a SPICE simulation of the active (or passive) network in order to determine the scaling factors
Leapfrog Topology

Design procedure: Elliptic design

• If the passive prototype has transmission zeros at the finite (elliptic filters), it has the form (low pass)

Before to describe the network and sketch the flow diagram it is worth to remove the bridging capacitors $C_2$ and $C_4$ through the use of the Thevenin’s theorem.

$$a_1 = \frac{C_2}{(C_1 + C_2)} \quad a_2 = \frac{C_2}{(C_3 + C_2)}$$
Leapfrog Topology

Design procedure: Elliptic design

- Similar modification follows if $C_4$ is removed

- The equations concerning the state variables $V_1$, $V_3$ and $V_5$ change:

\[
V_1 = \frac{1}{sC_1'}(l_5 - l_2) + \frac{V_3C_2}{C_1'} \quad C_1' = C_1 + C_2
\]

\[
V_3 = \frac{1}{sC_3'}(l_2 - l_4) + \frac{V_1C_2}{C_1 + C_3} + \frac{V_5C_4}{C_3 + C_4} \quad C_3' = C_2 + C_3 + C_4
\]

\[
V_5 = \frac{1}{sC_5'}(l_4 - l_6) + \frac{V_3C_4}{C_5'} \quad C_5' = C_4 + C_5
\]
Leapfrog Topology
Design procedure: Elliptic design

- After scalings, aimed to the active realization:
High order SC filter
Low-noise solution

- Inverse-Follow-the-Leader-Feedback

At low frequency only the noise $v_{n1}$ reaches the output node
Resonator pole frequency

- Let’s consider a two-integrator loop
- This is a building block for complex poles implementations
- Integrator finite dc-gain = $A_o$
- The poles are given by the roots of the denominator of $H_{\text{closed loop}}=1+H_1 \cdot H_2$

$$H_{res}(s) = \frac{\text{Forward Gain}(s)}{1+H_1 \cdot H_2} = \frac{FG(s)}{1+\left(\frac{A_o}{1+s \cdot A_o \cdot \tau}\right)^2} = \frac{FG^*(s)}{(1+s \cdot A_o \cdot \tau)^2 + A_o^2} = \frac{FG^*(s)}{s^2 \cdot A_o^2 \cdot \tau^2 + 2 \cdot s \cdot A_o \cdot \tau + A_o^2}$$

$$H_{res}(s) = \frac{FG^*(s)}{s^2 + s \cdot \frac{\omega_o}{Q_{res}} + \omega_o^2} = \frac{FG^*(s)}{s^2 + 2 \cdot \frac{s}{A_o \cdot \tau} + \frac{1}{\tau^2}}$$

$$\omega_o = \frac{1}{\tau} \quad Q_{res} = \frac{A_o}{2}$$

- The resonance frequency is given by the loop unity-gain frequency
- The integrator finite gain is limiting the maximum resonator $Q_{res}$
  - $Q_{res_{\text{ideal}}} = \infty$ only for $A_o = \infty$
Integrator quality factor

- The integrator transfer function is:

\[ H_{\text{int}}(s) = \frac{v_o}{v_i} = \frac{1}{R(\omega) + j \cdot X(\omega)} \]

- The integrator quality factor \( Q_{\text{int}} \) is defined as:

\[ Q_{\text{int}}(\omega) = \frac{X(\omega)}{R(\omega)} \]

- An equivalent definition is

\[ Q_{\text{int}} = \frac{1}{\phi_{\text{ideal}[\text{rad}]} - \phi_{\text{real}[\text{rad}]}} = \frac{1}{\Delta \phi[\text{rad}]} \]
Integrator quality factor

Example: Ideal integrator

\[ H_{\text{int}}(s) = \frac{1}{s \cdot \tau} \]

\[ Q_{\text{int}}(\omega) = \frac{\omega \cdot \tau}{0} = \infty \]

Example: Finite-gain integrator

\[ H(s) = \frac{A_0}{1 + s \cdot \tau \cdot A_0} = \frac{1}{1/A_0 + s \cdot \tau} \]

\[ Q_{\text{int}}(\omega) = \frac{\omega \cdot \tau}{1/A_0} = A_0 \cdot \omega \cdot \tau \quad \Rightarrow \quad Q_{\text{int}}(1/\tau) = A_0 \]
Resonator sensitivity

- In a resonator, the quality factor depends on the phase at the resonance frequency
  - The resonance frequency is the unity-gain frequency of the loop
    \[
    \frac{1}{Q_{\text{real}}} \approx \frac{1}{Q_{\text{ideal}}} + \delta \Phi_{\text{loop}}(\omega_0)
    \]

- A phase lead (\(\delta \Phi_{\text{loop}}(\omega_0) > 0\)) reduces the \(Q\)
  - LHS Poles at frequency lower than \(\omega_0\)
    - Opamp Finite dc-Gain

- A phase lag (\(\delta \Phi_{\text{loop}}(\omega_0) < 0\)) increases the \(Q\)
  - LHS poles at frequency higher than \(\omega_0\)
    - High-frequency non-dominant poles

![Graph showing phase and frequency response](image-url)
Resonator sensitivity

Amplitude and phase: Single non-dominant pole

- Each integrator presents a phase shift due to a dc-gain (pole at $f_p$)
  \[
  \delta \varphi(f) = - \arctan(f / f_p) \approx -f / f_p
  \]

- A two-integrator loop has a phase shift given by
  \[
  \delta \varphi_{tot}(f) = 2 \cdot \delta \varphi(f) = -2 \cdot f / f_c
  \]

- The two-integrator loop nominal Q ($Q_o$) deviates to a real value:
  \[
  \frac{1}{Q_{\text{real}}} \approx \frac{1}{Q_o} + 2 \cdot \delta \varphi(f_o) = \frac{1}{Q_o} - 2 \cdot f_o / f_p
  \]
  \[
  \frac{\delta Q_o}{Q_o} \approx 2 \cdot Q_o \cdot \frac{f_o}{f_p} < \left[ \frac{\delta Q_o}{Q_o} \right]_{\text{specs}} \Rightarrow f_p > \frac{2 \cdot Q_o \cdot f_o}{\left[ \frac{\delta Q_o}{Q_o} \right]_{\text{specs}}}
  \]

For example: $f_o=10$MHz, $Q_o=10$, $[\delta Q_o/Q_o]_{\text{specs}}<10\%$  \Rightarrow  $f_p > 2$GHz
Resonator sensitivity

Amplitude and phase error: Multiple singularities

Ideal

- Dom. Pole at the origin
- Infinite dc-gain
- No secondary poles
- $90^\circ$ phase at $\omega_0$

$$Q_{\text{int}}^{\text{ideal}} = \infty$$
$$\frac{1}{Q_{\text{int}}^{\text{ideal}}} = 0$$

- The dominant pole gives a phase lead at $\omega_0$
- The secondary poles give a phase lag at $\omega_0$

Real

- Dom. Pole at $\alpha/\omega_0$
- $\alpha$ dc-gain
- Some secondary poles $p_i$
- $(90^\circ + \Delta\phi)$ phase at $\omega_0$

$$Q_{\text{int}}^{\text{real}} = \frac{1}{\alpha} - \sum_{i=2}^{\infty} \frac{1}{p_i}$$
Resonator sensitivity
Amplitude and phase error effects

- Phase lag and phase lead could compensate
  But they depend on different parameters
  A high $Q$-sensitivity results

- The best solution is move the singularities as far as possible from $\omega_0$
- The error in the passband is proportional to $Q_{\text{filter}}/Q_{\text{intg}}$
Integrator phase values

- Integrator phase

<table>
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<th>$f_o/f_p$</th>
<th>Phase Error [°]</th>
<th>$Q_{int}$</th>
<th>$f_o/f_p$</th>
<th>Phase Error [°]</th>
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</tr>
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</table>
Implementation of the Designed Flow-graph

• There are several possibilities to implement the designed flow-graph:
  
  Passive
  - RC or LC
  
  Active
  - Active-RC
  - MOSFET-C
  - Gm-C
  - Gm-Active-RC
  - Active-Gm-RC
  - Switched-capacitor
  - Switched-current
  ........
SC filters are intrinsically tuned (Slave to the clock)
Continuous-Time filters (Gm-C, LC) need continuous or discrete tuning
Analog Filters Scenario

Different techniques comparison (1998)

- LC filters
- gm-C filters
- SC filters (ΣΔ modulator)

It is running with technology scaling
Comparison of Filtering Techniques

Main possible alternatives

- Sampled Domain
  - Switched Capacitor
  - Switched Current

- Continuous Time
  - gm-C (open loop approach)
  - MOSFET-C (closed loop approach)
  - Active-RC
  - L-C

- Intrinsic precision is obtained
- Key problem is turn-on pass transistor switches
- If switches can be turned on rail-to-rail operation can be achieved
- Needs to be tuned to have absolute precision
- Rail-to-Rail swing sometimes difficult
Different Continuous-Time Integrators

**Active-RC**
- Good Linearity even at low voltage
- Small number of active elements
- Insensitive to parasitic
- It must drive low impedance loads
- No continuous time tuning mechanism
- Limited speed

**MOSFET-C**
- Insensitive to parasitic
- Small number of active elements
- It must drive low impedance loads
- Extra singularities due to the opamp
- Small swing at low voltage due to tuning
Different Continuous-Time Integrators

**Gm-C-Opamp**
- Insensitive to parasitics
- It has not to drive low impedance loads
- Small output swing transconductors
- Largest number of active elements
- Extra poles i.e. not very fast

**Gm-C**
- Fastest of all possible approaches
- It has not to drive low impedance loads
- Sensitive to parasitic capacitance
- Large input and output signals
- Larger number of active elements
High-Order Active Filter

Active-Gm-RC

- Low-sensitivity to parasitics
- Largest number of active elements
- Extra poles i.e. not very fast

Switched-Capacitor

- Large input and output signals
- Insensitive to parasitic capacitance
- Larger number of active elements
- Clock needed & System complexity